Ising Models
and Contagion Pricing

Simulation and Phenomenology

Overview

Methods

- Random Variables
- Metropolis-Hastings Algorithm
- Markov Chain Monte Carlo Simulations
- Boltzmann's Statistical Mechanics
- Spin Models

Results

- Ising Spin Simulations
- Ising Small-World Simulations

Future Work and Current Research

Acknowledgements and Links

Random Variables

A **RV** can be seen as a “set” S and a map P from S to the Reals...

sum([for every s in S, P(s)]) = 1

(and some other requirements).

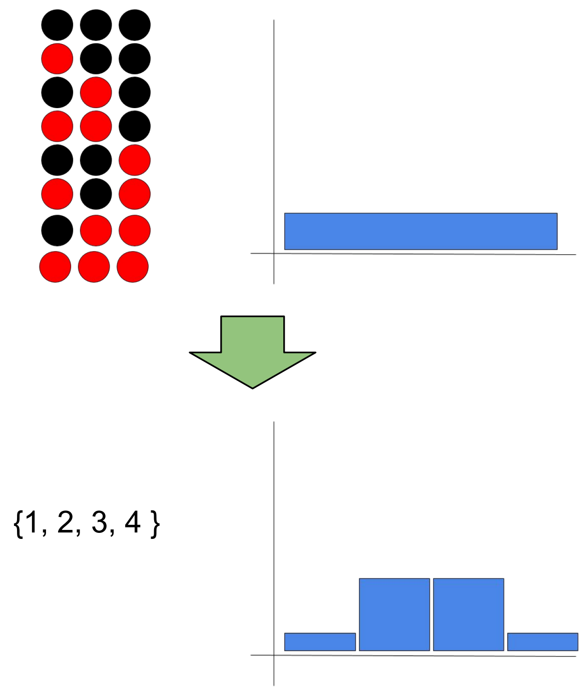
A **statistic** F can be seen as function that takes an RV and maps to a **new RV**...

F (f, (S, P)) := (S’, P’) **where**

S’ = Range of f

P’(s’) := sum([for every s in S s.t. f(s) = s’])

(and some other transformations).

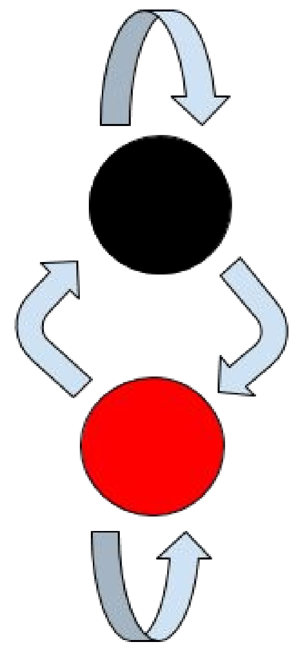


Metropolis-Hastings

How do we take a RV and sample values that "look" like they came from the RV? A histogram of these samples better look like the P defined in the RV. Take the probability to be...

$$p(a \to b) = \min\left[1, \frac{\pi(b)}{\pi(a)}\right]$$

If we do this, the simulated data and histograms will "look" like P.



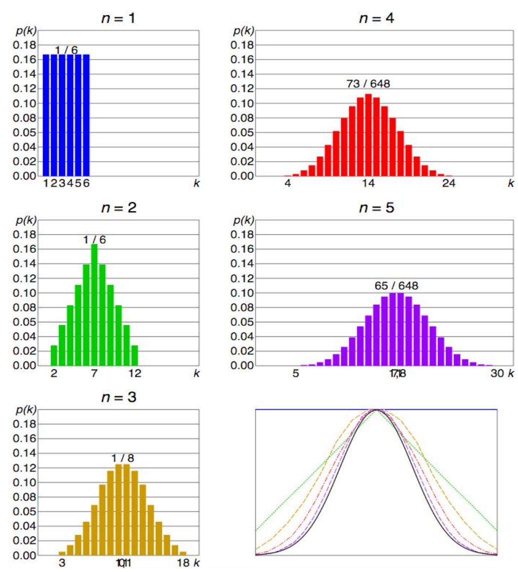
Markov Chain Monte Carlo Simulations

All about...

f(x) - a function of x, which is a RV

f(x) may not have a closed form solution, so how do we compute the distribution of f(x), without using any calculus?

1. Simulate x
2. Compute f(x) at each step
3. Show the distribution of f(x)



Boltzmann's Assumption

Measure some values of energy, and take the average <E>.

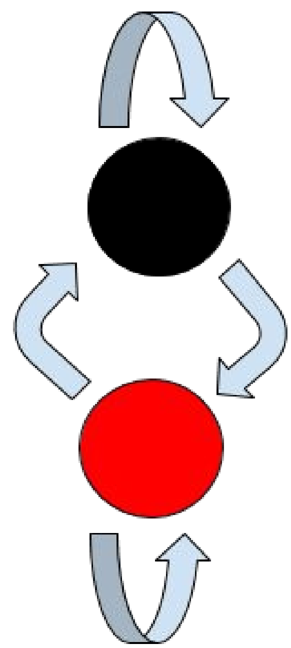
So you have a histogram of E, so how to we get a P, or better put, a probability distribution?

Take...

$$P(s) = \exp(E / kT)$$

But remember, transition probabilities using MCMC are...

$$P(a \to b) = \min(1, P(b) / P(a))$$



Spin Models

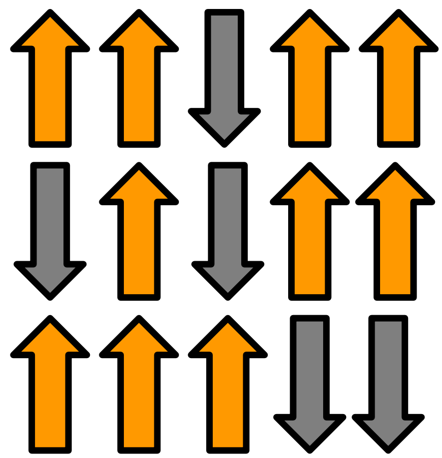
Define an energy of a lattice to be...

E = sum([x * y for all neighboring spins x and y])

We can simulate a system using Boltzmann's Assumption

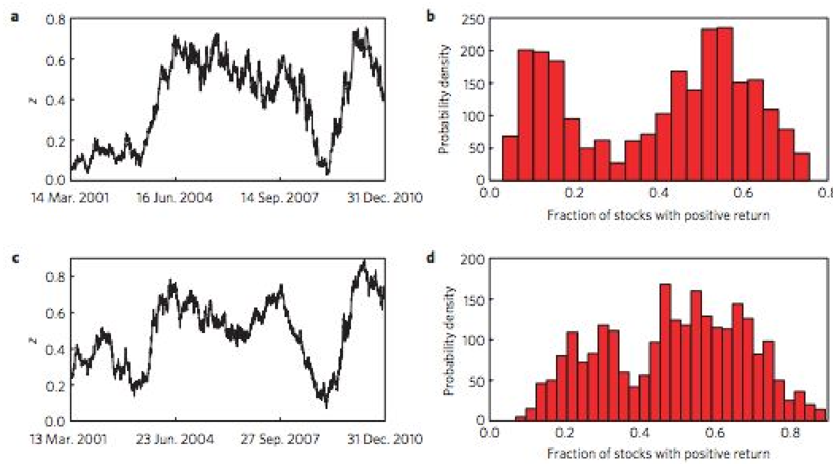
What kinds of lattice can we use?

What exactly is a "neighbor"?



Future Work and Current Research

S&P 500 stock data shows bimodal distributions just like ising models on small world networks. Maybe stock data isn't driven by a model at a "critical point", but simply a small world network. Researchers are currently simulating prices using small world networks.



Acknowledgements and Links

Biondo A., Pluchino A., Rapisarda A., (2015) *Modelling Financial Markets by Self-Organized Criticality*, arXiv:1507.04298v2 [source]

Chib S., Greenburg E., (1996) *Understanding the Metropolis-Hastings Algorithm*, The American Statistician, Vol. 49, No. 4., pp. 327-335 [source]

Jaynes, (1957) *Information Theory and Statistical Mechanics*, The Physical Review, Vol. 106, No. 4, 620-630 [source]

Krauth W., (2006) *Statistical Mechanics: Algorithms and Computations*, Oxford University Press Inc., New York

Majdandzic A., Stanley E., et. al, (2014) *Spontaneous Recovery in Dynamical Networks*, Nature Physics, Vol. 10 34-38 [source]