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Experimental econophysics: Complexity, self-organization, and emergent properties

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ABSTRACT

Experimental econophysics is concerned with statistical physics of humans in the laboratory, and it is based on controlled human experiments developed by physicists to study some problems related to economics or finance. It relies on controlled human experiments in the laboratory together with agent-based modeling (for computer simulations and/or analytical theory), with an attempt to reveal the general cause–effect relationship between specific conditions and emergent properties of real economic/financial markets (a kind of complex adaptive systems). Here I review the latest progress in the field, namely, stylized facts, herd behavior, contrarian behavior, spontaneous cooperation, partial information, and risk management. Also, I highlight the connections between such progress and other topics of traditional statistical physics. The main theme of the review is to show diverse emergent properties of the laboratory markets, originating from self-organization due to the nonlinear interactions among heterogeneous humans or agents (complexity).

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1. Introduction

It might be a type of human inability that a single scientist cannot research on all the aspects of the nature and society. Owing to the human inability, science has been divided into many disciplines, e.g., mathematics, physics, chemistry, biology, economics/finance, and so on. As a result, specific researchers always work in the field of a specific discipline. For example, the researchers, under the name of physicists, work in the specific field of physics. After a longtime separation between physics and economics/finance, now the time is ripe for their integration, so that they could help each other to develop, at least to some extent.

1.1. Physics needs economics/finance

If one counts from G. Galilei (Feb. 15, 1564–Jan. 8, 1642), physics, the study of nature, has developed for 400 years or so. This duration is not long, compared with millions of years for which humans have lived on earth. But, everyone has witnessed the significant changes of human life brought by physics, say, electricity, computers, mobile phones, artificial satellites, utilization of nuclear energy, and so one. All of these changes are an outcome of physical knowledge of the natural world. According to this fact, it is no doubt that the ideas and methods of physics are useful to handle the natural world. Here the natural world means it contains non-intelligent units, which have no adaptability due to the lack of learning ability. For example, such non-intelligent units are electrons, atoms, molecules, colloidal particles, and so on. In sharp contrast to the natural world, the social world is full of intelligent units, which have adaptability due to the existence of learning ability. For instance, such intelligent units involve humans, companies (containing many humans), countries, and so on.

Science is always driven by curiosity, and physics is no exception. Inspired by the success of physics in handling the natural world, one might curiously ask whether the ideas and methods originally developed from physics for treating the natural world are also useful for the social world. The answer is definitely positive. But, the reader might continue to ask: "what do you mean by saying the ideas and methods originally developed from physics for treating the natural world?" or "what are physical ideas and methods?"

(1) Two physical ideas

a. Reasons extracted should be coarse-grained

Let me take the freely falling object as an example. The number of reasons determining falling height could be up to N: time, air resistance, atmospheric pressure, humidity, etc. However, G. Galilei (Feb. 15, 1564–Jan. 8, 1642) neglected the $N-1$ reasons, and considered only the relation between falling height (h) and time (t), yielding $h=(1/2)gt^2$. Here g is acceleration (a constant). As a result, he established the law of free fall, which helped I. Newton (Dec. 25, 1642–March 20, 1726) to successfully establish classical mechanics in the discipline of physics. According to this law, the first idea of physics comes to appear: one should extract crucial reasons, or equivalently *reasons extracted should be coarse-grained*.

b. Results obtained should be universal

After Galilei's $h=(1/2)gt^2$, I. Newton (Dec. 25, 1642–March 20, 1726) revealed his second law, $F=ma$, where F is force, m is mass, and a is acceleration. This second law helps to explain not only the freely falling object on the earth (by setting $a=g$ and seeing F as gravity), but also the planetary motion in the sky (that had been empirically summarized in the laws of planetary motion by J. Kepler (Dec. 27, 1571–Nov. 15, 1630)). Besides, Newton's second law can even be used to predict new phenomena. Say, on Aug. 31, 1846, U. Le Verrier (March 11, 1811–Sep. 23, 1877) first predicted the existence and position of Neptune by using Newton's second law plus Newton's law of gravity; Neptune was subsequently observed on Sep. 23, 1846 by J. G. Galle (June 9, 1812–July 10, 1910) and H. L. d'Arrest (Aug. 13, 1822–June 14, 1875). The success of Newton's second law indicates the second idea of physics is that "results obtained should be universal". Here, the "universal" means that the results should not only help to explain the existing phenomena, but also help to predict the future or unknowns.

(2) Three physical methods

a. Empirical analysis

From Aristotle (384–322 B.C.) to J. Kepler (Dec. 27, 1571–Nov. 15, 1630), physicists first observed the natural world, and then analyzed the observations, yielding many empirical results, say, Kepler's laws of planetary motion. Such analysis is just empirical analysis, which is based on existing data in nature.

Advantages of empirical analysis: reliability and huge data. Here, the "reliability" means that according to the data collected from the nature itself, any results obtained from the data should be reliable; the "huge data" means that the number of data in nature is huge, which is definitely helpful for understanding the natural world.

Disadvantages of empirical analysis: uncontrollability (correlation) and non-formatting. Since the data are collected from the nature, they are always uncontrollable. Then, what empirical analysis can produce is mainly correlations, instead of causations. Clearly, the causations represent a deeper understanding than the correlations. Regarding "non-formatting", it is easy to understand: the format of data existing in nature is not fixed, but dependent on how people to collect them. The non-formatting of data causes trouble for people to investigate.

b. The combination of empirical analysis and controlled experiments

In general, empirical analysis helps to reveal correlations, rather than causations. Inspired by empirical analysis, G. Galilei (Feb. 15, 1564–Jan. 8, 1642) started to perform experiments in the laboratory by purposefully tuning one or a few parameters/conditions (but all the other parameters/conditions are fixed), in order to reveal cause–effect relationships (causation). His method is based on controlled experiments.

Advantages of controlled experiments: controllability (causation) and formatting. These are just the inverse of the above-mentioned disadvantages of empirical analysis. Such experiments are controllable because one can tune one variable and see its effect (causation). Regarding "formatting", it means the format of data could be conveniently organized during the experiment.

Disadvantages of controlled experiments: deviations and few data. Since such experiments are conducted in the laboratory, the experimental data may be different from their counterparts in nature. The difference is just what we mean "deviations". On the other hand, the experimental data produced in the laboratory cannot be huge, as one can easily imagine. Thus, I indicate "few data" herein.

c. The combination of empirical analysis, controlled experiments and theoretical analysis

Due to the above-mentioned advantages and disadvantages of either empirical analysis or controlled experiments, I. Newton (Dec. 25, 1642–March 20, 1726) combined both empirical analysis and controlled experiments. For instance, when he explained Kepler's laws of planetary motion (outcome of empirical analysis), he also explained Galilei's law of free fall (outcome of controlled experiments). The combination of both empirical analysis and controlled experiments reserves their advantages, but removes their disadvantages. More importantly, Newton also realized that the combination of both empirical analysis and controlled experiments can produce results only for specific areas: empirical analysis corresponds to the specific objects producing empirical data (e.g., Kepler's laws of planetary motion are only valid for planets); controlled experiments are related to specific laboratory samples/devices producing experimental data (say, Galilei's law of free fall specifically holds for the freely falling object in the laboratory). As a result, Newton utilized theoretical analysis (based on mathematics like calculus) to generalize the results (obtained from the combination of both empirical analysis and controlled experiments) from specific areas to broad areas. For example, his second law ($F=ma$) can help to explain not only the motion of either planets (described by Kepler's laws of planetary motion) or freely falling objects (described by Galilei's law of free fall), but also the motion of many other objects including a single molecule. Owing to the unprecedent success of this generalization (which is proved by the fact that physics has significantly improved our human life), the method of combining empirical analysis, controlled experiments and theoretical analysis has become the fundamental method for developing physics. Certainly, in reality, it is already enough for achieving some excellent results by using only one or two of empirical analysis, controlled experiments and theoretical analysis. This fact depends on specific topics, for example, in modern condensed matter physics, where empirical analysis is hardly used. But, in modern astrophysics, controlled experiments are rare. It is not necessary for me to go into details here. In principle, the above mentioned combination is an ideal, complete method.

So far, I have completed answering the question "what are physical ideas and methods?".

Last but not least, even though physics has helped to significantly improve human life due to the deep understanding of the natural world by using the above-mentioned physical ideas and methods, it might be not suitable for people to immediately expect too much when physical ideas and methods are used to understand the social world. Why? Please note many of the above-listed applications brought by physics are not a direct purpose of original research. For example, when M. Faraday (Sep. 22, 1791–Aug. 25, 1867) discovered the law that magnetism is able to produce electricity, he did not know whether it is genuinely useful for human life. The reason why he did the research was only due to curiosity. Actually, in history, the large-scale application of electricity only started at the end of the 19th century when Faraday had passed away for many years. This means people must be patient to wait for the application of a physical discovery since physicists must take time to study.

In a word, the reason why physics needs economics/finance lies in curiosity of physicists, which may broaden the realm of physics, especially statistical physics.

1.2. Economics/finance needs physics

Briefly speaking, economics is a discipline on how to allocate scarce resources efficiently. Since A. Smith (June 5, 1723–July 17, 1790), economics has developed for more than 200 years. In the duration, mathematics has been introduced into economics, thus causing economics to be a quantitative discipline already. Because the object discussed in the field of economics is human-activities-related phenomena in the social world, which are too complex, economics is still far from perfect. For example, existing economic theories failed to envisage even the possibility of a financial crisis like the latest one [1]. Thus, economics needs different ideas or methods in an attempt to perfect itself. Physics may be a candidate discipline for economics to absorb such ideas and methods. In this sense, economics needs physics, so that people might scrutinize some economic problems from a different perspective, thus yielding different insights. Certainly, economics can also resort to ideas or methods being beyond physics, e.g., evolutionary biology.

The above conclusion holds for finance, too. Regarding finance, its relation with economics is similar to the relation between applied physics and basic physics. That is, finance focuses on application research, but economics focuses on basic research. Say, if a seller sell me a pen at a price of 1 Chinese Yuan, the exchange behavior between me and the seller belongs to finance, but the reason why the pen costs 1 Chinese Yuan rather than 100 Chinese Yuan belongs to economics. Nevertheless, throughout this review article, I shall not separate economics and finance distinctly because both of them are closely related to trading behaviors of human beings.

1.3. Combining physics and economics/finance gives birth to econophysics

Physics meets economics or finance, giving birth to econophysics; the word "econophysics" first appeared in the published article by H. E. Stanley et al. as early as 1996 [2]. According to the above introduction, econophysics is a new branch of physics (at least in the eyes of physicists like me), which uses physical ideas and methods (listed in Section 1.1) to analyze some problems related to economics or finance. Loosely speaking, econophysics is what physicists do in the field of economics or finance since these physicists are naturally weaponed with physical ideas and methods. Incidentally, researches within the scope of econophysics actually appeared much earlier than the mid-1990s (e.g., Ref. [3]), but at that time, the word "econophysics" was not yet coined.

To briefly summarize what I have mentioned above, the aim of econophysicists could be at least two-folded: one is to broaden the realm of traditional physics (throughout this review article, the phase "traditional physics" means the physics that is used to study the nature with non-intelligent units like atoms, rather than the society with intelligent units like humans), especially statistical physics, the other is to scrutinize some economic or financial problems from a physical perspective.

1.4. Empirical econophysics versus experimental econophysics

In general, econophysics contains three methods: empirical analysis (starting from the articles by H. E. Stanley and his coworkers in the mid-1990s, see for example Refs. [4,5]), controlled experiments (starting as early as 2003 by T. Platkowski and M. Ramsza [6]), and theoretical analysis (pioneered by Y. C. Zhang and his coworkers in the mid-1990s, e.g., see Ref. [7]). Here, empirical analysis in the field of econophysics is based on existing data in real markets, and controlled experiments mean experiments conducted in the laboratory, which produce data by tuning one (or few) variable(s)/condition(s). The theoretical analysis in the field of econophysics is based on agent-based modeling (or system modeling), which has two approaches: agent-based simulations (also called computer simulations) and analytical theory. Agent-based simulations have helped to develop econophysics significantly [8,9], which are an analogue of molecular dynamics simulations [10], Monte Carlo simulations [11], or finite element simulations [12,13] in traditional physics. According to traditional physics, for analytical theory, one needs to start from some common laws or principles (e.g., see Ref. [14,15]), which often lack in social human systems (the research object of econophysics). Thus, compared with agent-based simulations, analytical theory has not played the same important role in econophysics.

Since the birth of econophysics in the mid-1990s [2], empirical analysis has dominated the research community of econophysics till now, which forms a branch of econophysics, namely, empirical econophysics. But, as a discipline, econophysics is still too young, which has vast development space. As one knows, the maturity of traditional physics is mainly due to the role of controlled experiments in the laboratory. Accordingly, it seems unbelievable that in the future, econophysics without controlled experiments could be as mature as traditional physics. Thus, I believe that for developing econophysics more healthily, econophysicists must resort to controlled experiments. In this sense, for the time being, in order to emphasize the importance of controlled experiments, I suggest a different branch in econophysics, i.e., experimental econophysics, which mainly focuses on controlled experiments in the laboratory. Accordingly, in this review article, econophysics is divided into two branches: one is empirical econophysics, the other is experimental econophysics.

Clearly, the above two branches are divided according to the two different methods. It is worth noting that in the above paragraph, I did not mention theoretical analysis. But, in the eye of physicists like me, these simulations must be used to understand either empirical data (in real markets) or experimental data (in laboratories). So, for understanding empirical data, theoretical analysis is a supplementary tool in empirical econophysics. On the other hand, for mainly understanding experimental data, theoretical analysis serves as an additional tool in experimental econophysics.

The focus of this review is on experimental econophysics. Regarding empirical econophysics, several leading experts have published a couple of excellent monographs [5,8,9,16] and reviews [17,18]. If the reader can read Chinese, I would also like to recommend him/her to read two relevant monographs in Chinese [19,20].

1.5. What is the methodology of experimental econophysics

Section 1.1 describes the combination of empirical analysis, controlled experiments and theoretical analysis, which are fundamental in traditional physics. Owing to the great success of this combination method in traditional physics, I believe experimental econophysics should also follow this combination method (which started from the article by Wang et al. in 2009 [21]). For this purpose, one should do according to the following three steps:

- Step 1: He/she should either do empirical analysis or survey the literature on empirical data in real markets. Obtain or find empirical results.
- Step 2: Then, he/she considers how to design a controlled experiment in the laboratory. In the mean time, he/she should keep the above-mentioned physical idea ("Reasons extracted should be coarse-grained") in mind, and extract the key factor(s) which affects the empirical results. Perform controlled experiments, revealing cause–effect relationships between the factor(s) and the above empirical results.
- Step 3: Perform theoretical analysis (agent-based simulations and/or analytical theory), in order to extend the cause–effect relationships obtained in Step 2 beyond the specific experimental limitations (namely, specific subjects, specific avenue, and specific time). Use the relationships to explain existing phenomena (empirical results) and predict the future or unknowns. In other words, until the end of Step 3, the final results (relationships) should be universal enough, which echoes with the above-mentioned second physical idea ("Results obtained should be universal").

The above three steps serve as the complete, ideal methodology of experimental econophysics. But, here I should remark that some projects in econophysics cannot strictly follow the three steps (at least for the time being) due to the complexity of real human systems like stock markets; see for example Section 3.

1.6. Complexity, self-organization, and emergent properties

As indicated in the title of this review, I shall survey complexity, self-organization, and emergent properties in the field of experimental econophysics. For clarity, here I had better explain more about the three key words: *complexity*, *self-organization*, and *emergent properties*. It is worth noting that the below explanations are specifically given for this review only.

Firstly, the *complexity* is used to characterize a system with heterogeneous humans or agents, where they have nonlinear interactions. As a result, the property of the whole system cannot be inferred from the individual humans or agents without nonlinear interactions. In other words, the *complexity* is strictly related to the nonlinear interactions among heterogeneous humans or agents.

Secondly, regarding the *self-organization*, I prefer to ask for help from Wikipedia (http://en.wikipedia.org/wiki/Self-organization); details are as follows (with minor revisions). Self-organization is a process where some form of global order or coordination arises out of the local interactions among the humans or agents of an initially disordered system. This process is spontaneous: it is not directed or controlled by any human or agent inside or outside the system; however, the laws followed by the process and its initial conditions may have been chosen or caused by a human or an agent (this fact is similar to traditional physics, where an external electric field may be exerted to produce new phenomena; for example, see Ref. [22]). It is often triggered by random fluctuations that are amplified by positive feedback. The resulting organization is wholly decentralized or distributed over all the humans or agents of the system. As such it is typically very robust and able to survive and self-repair substantial damages or perturbations.

Thirdly, emergence is a process where larger regularities or patterns arise through nonlinear interactions among the humans or agents that themselves do not exhibit such properties. Here "larger regularities or patterns" are just *emergent properties* to be surveyed in this review.

In summary, the main theme of the review is to show diverse *emergent properties* of the laboratory markets (belonging to a kind of complex adaptive systems), originating from *self-organization* due to the nonlinear interactions among heterogeneous humans or agents (*complexity*).

2. Basic knowledge

This section presents some fundamental knowledge or background, which may help to understand the forthcoming sections.

2.1. Hayek hypothesis: A pragmatic hypothesis

Even nowadays, the typical method to study economic problems is the hypothetical-deductive method. Economists often derive an optimal situation of a system from certain assumptions, such as the complete knowledge of a preference system or information. F. A. Hayek (May 8, 1899–March 23, 1992; 1974 Nobel prize winner in economic sciences) pointed out in his famous thesis "The Use of Knowledge in Society" published in 1945 [23] that this was totally a misunderstanding of social problems because no one could simply acquire the entire data of such assumptions. So despite the allocation problem under certain assumptions, a more important problem was how to obtain and use the decentralized resources and information.

Another thing economists always neglect is the specific knowledge of individual. Other than scientific knowledge, this specific knowledge only gives its owner a unique benefit due to his/her own understanding of people, environment and other special circumstances. That is the exact part of knowledge economists are used to put into the assumptions which is equally important as scientific knowledge. As the comparative stability of the aggregates cannot be accounted for by the "law of large numbers" or the mutual compensation of random changes and the fact that the knowledge of this kind which by its nature cannot enter into statistics, all plans should be made by the "man on the spot" rather than any central authorities.

Since the central authorities are limited, will the plans made by individuals reach a so-called equilibrium state? Hayek's answer was "yes". If all individuals follow the simple regulation of "equivalence of marginal rates of substitution", which is the basic of microeconomics, the market will indeed be in an equilibrium state finally, without the necessity of the knowledge of the entire market. Under the "magical" market mechanism of price, once someone finds an arbitrage opportunity of a commodity, the price of this commodity will change. Thus the marginal rates of substitution of this commodity to other commodities change, causing another round of price change. This effect spreads to more and more kinds of commodities and gradually covers the whole market, though maybe no one knows why such changes happen. The whole acts as one market, not because any of its members survey the whole field, but because their limited individual fields of vision sufficiently overlap so that through many intermediaries the relevant information is communicated to all.

Actually, the price system is just what A. Smith (June 5, 1723–July 17, 1790) called "the invisible hand" [24], a mechanism for communicating information, and the most significant fact about this system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action. Even if people know about all the factors of a commodity, the actual price is not available unless obtained from a market with price system.

The above content has been known as the Hayek hypothesis [23], which asserted that markets can work correctly even though the participants have very limited knowledge of their environment or other participants. Certainly, traders have different talents, interests and abilities, and they may interpret data differently or be swayed by fads. However, there is still room for markets to operate efficiently.

In 1982, "the father of experimental economics" V. L. Smith (2002 Nobel prize winner in economic sciences) [25] tried proving the Hayek hypothesis using 150–200 experiments under different circumstances which he thought a correct method to select a reliable theory. The trading behavior of the market participants led the market to a competitive equilibrium under a double auction regulation without any extra information (the participants, i.e., subjects, only knew their own value of the commodity and the market price), the result of which was contrary to the classical theory of price taking hypothesis and complete knowledge hypothesis.

A key characteristic of controlled experiments was its specific convertible supply and demand condition and the reward system to stimulate the subjects. Once the supply and demand are determined, the equilibrium market price is also determined and whether the market was operated well could be easily observed. Although all the experiments [25] had different subjects and supply and demand conditions, they all end with the equilibrium state no matter whether in a stationary or dynamic environment. Even though the experiments [25] were still not perfect, a reliable result related to the Hayek hypothesis was that the attainment of competitive equilibrium outcomes is possible under much less stringent conditions.

To sum up, the Hayek hypothesis helps to account for the reliability of controlled experiments that contain finite subjects. Thus, this Hayek hypothesis may serve as a theoretical foundation of experimental economics [26] and experimental econophysics (introduced in this review article), both of which are based on controlled human experiments in the laboratory. Besides, in the field of experimental econophysics, theoretical analysis also helps to validate and generalize results obtained from controlled experiments; more details can be found in Sections 1.1 and 1.5.

2.2. How to design controlled human experiments

In this review article, I shall introduce the latest research progress on controlled experiments in the field of experimental econophysics. As a source of econophysics research ideas, the method of controlled experiments has become increasingly important in relative studies. Controlled experiments have various ways to conduct. For example, in a simple experiment based on the minority game [7], the organizer may require subjects to close their eyes and raise hands to signal their choices between the two rooms in the game. Here, closing eyes prevents communications among the subjects so that it can be guaranteed that they make decisions independently. However, when regulations of an experiment become more complicated, or when the number of subjects becomes larger, computer-aided controlled experiments begin to show their efficiency as they can implement any experimental design easily and can also collect experimental data and reveal real-time statistical results quickly.

A computer-aided controlled experiment needs the programming of both the server and the client (Fig. 1). Two primary missions of the client are to distribute related information to the users and also to provide users a channel to upload their own personalized choices. And the server's main tasks are to store users' information, to process users' uploaded data and to generate new information based on feedbacks from the behaviors of users. Generally, the client may be designed in the form of web pages in order to reduce costs and increase scalability. Any computer with a network connection can be easily set as a client. The servers of all the experiments in Sections 3–8 were constructed on the architecture of Linux + Apache + MySQL + PHP/Python. (Readers can download a source code example from the link: http://t.cn/zOlkLEk.)

Fig. 1 shows a general schematic flow chart abstracted from the various experimental systems which will be introduced in Sections 3–8. Actually, the detailed designs of various systems are different in many aspects. For example, in the experiment of controlled laboratory stock market (Section 3), the server has to process every new order immediately since the time in the experiment is continuous. The herd experiment in Section 4 needs to add some robot agents (produced by a computer program) when computing the final outcome. The experiment of risk and return in Section 8 requires every subject to set their initial investment ratio, and the final outcome is not simply the "win" or "lose" but the final returns. Hence, we need to make minor revisions of the server and the web page accordingly for each specific system.

Fig. 2 depicts some screenshots of web pages in a certain controlled experiment. And Fig. 3 shows a site photo in a controlled experiment.

2.3. El Farol bar problem versus minority game

Here we first present the real El Farol bar problem [27], which gave birth to the minority game [7] discussed in Sections 4–6. An early summary of minority games can be found in the book by Challet et al. [9].

2.3.1. El Farol bar problem

The essence of formation of human social activities lies in the acquisitiveness for resources. In many social and biological systems, the agents always spontaneously adaptively compete for limited resources, and thus change their environments. In order to effectively describe the system with the complexity, scientists have made a series of attempts. Such a resource competition system is just a kind of complex adaptive system.

For economic systems, the basic issue appears as well. Generally, in an economic market, if the resources are rationally allocated, the market is full of vitality. Otherwise, the development will be impeded, at least to some extent. Thus, the allocation of resources is the most fundamental economic problem. As one knows, most of popular economics theories are related to deductive reasoning. According to those economic theories, as long as all individuals are almost smart, everyone will choose the best action, and then each individual can reason his/her best action.

However, people gradually find that in the real life, individuals often have no complete rationality and superb deductive reasoning ability when making decisions. Instead, it is very common for them to simply use the feasible method of trial and error. Therefore, it looks like inductive generalization and continuous learning when real individuals make decisions (namely, inductive reasoning).

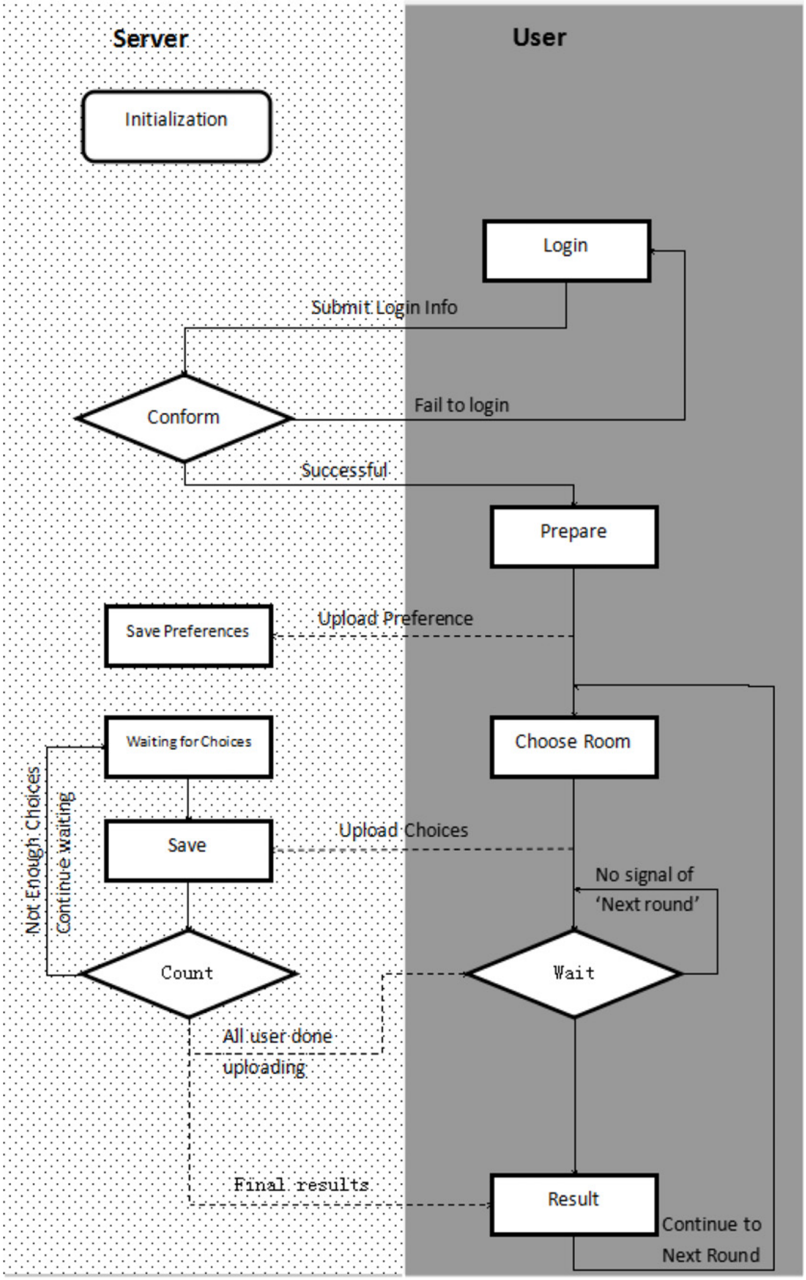

Fig. 1. A schematic flow chart showing how to compile the computer program for conducting controlled experiments.
Source: Adapted from Ref. [20].

In the game theory, researchers often use evolutionary games to study the similar dynamic process. However, when using evolutionary game models, economists usually do not take into account the character of limited rationality. Therefore, they cannot convincingly yield interesting phenomena and critical phase transition behaviors. A social human system contains a large number of agents who have the limited ability of inductive reasoning. Even so, the microscopic simplicity can still lead to the complexity of the macroscopic system. Obviously, from a physical point of view, this system has a variety of statistical physical phenomena.

In the past, there were some studies about the allocation of resources. For example, in 1994, economist W. B. Arthur put forward a very representative resource allocation problem, the El Farol Bar problem, when he studied the inductive reasoning and bounded rationality [27]. It can be described as follows.

There is an El Farol bar in Santa Fe (a city in New Mexico of United States) which offers Irish music on every Thursday night. Each Thursday, 100 persons (here the 100 is only set for concreteness) need to decide independently whether to go

Fig. 2. Web pages of a controlled experiment: (a) background management interface, (b) login, (c) login successful, (d) choose a room, (e) waiting for the result, and (f) result.
Source: Adapted from Ref. [20].

to this bar for fun or stay at home because there are only 60 seats in the bar. If more than 60 persons are present, the bar is so crowded that the customers get a worse experience than staying at home. If most people choose to stay at home at that day, then the people who go to the bar enjoy the elegant environment and make a wise choice.

In this problem, Arthur assumed no communication in advance among the 100 persons. They only know the historical numbers in the past weeks and have to make decisions independently. In order to make a wise choice, each person needs to possess his own strategies which are used to predict the attendance in the bar this week. People cannot obtain the perfect equilibrium solutions at initial time when making decisions. They must consider others' decisions, and keep learning according to the limited historical experience in their mind. The elements of inductive reasoning and limited rationality in the El Farol bar problem lay a foundation for the further development of modeling in econophysics, as shown in Section 2.3.2.

Fig. 3. A site photo in a controlled experiment organized by my group on September 27, 2013.

2.3.2. Minority game

Inspired by the above El Farol bar problem, physicists D. Challet and Y. C. Zhang in 1997 proposed a minority game to quantitatively describe this problem and statistically analyzed the emerging collective phenomena in complex adaptive systems [7]. In the following years, scientists have done extensive researches about the minority game and its applications in different fields, which have significantly promoted the development of econophysics [8,9]. We introduce the minority game model as follows.

There are two rooms (indicated as Room A and Room B) and N agents, where N is an odd number. Each agent chooses independently to enter one of the two rooms. If one room contains fewer agents than the other, then the agents in this room win. That is to say, the minority wins. The two rooms in the minority game actually correspond to the case of unbiased distribution of two resources. This game is repeated. Each agent can only make a decision next time according to the historical information. As a matter of fact, in the daily life, people often face similar choices. Examples include choosing which road to avoid traffic jam during rushing hours and choosing a less crowded emergency exit to escape. Although each of us can keep learning from limited historical experiences, it cannot guarantee that we make a correct choice every time.

In the minority game, the decision-making process which is based on the historical information is modeled to form strategy tables. For the minority game, one assumes that agents' memory length of the historical information is limited. Each agent can only remember the latest m rounds. If $m = 2$, it can form a strategy as shown in Table 1. The historical information in the left column records the attendance in the past two rounds, which is filled with a string of bits of 0 and 1. For example, a string of "10" represents the past two winning rooms, Room A and Room B. The right column is the prediction which is filled with bits of 0 or 1. Bit 1 is linked to the choice of Room A for entrance, while bit 0 to that of Room B. So one can obtain a strategy pool with a size of 2^{2^m}. As m increases, the total number of strategy tables increases rapidly. In the original minority game model, the designers let each agent randomly select strategy tables. That is, the right column of each strategy table is randomly filled with 0 or 1. These agents are likely to repeat the same selected strategy (namely, the right columns of the strategy tables are the same). However, appropriately increasing the memory length can significantly reduce the repetition probability. Here it is worth noting a special case: if the right column of a strategy table is all 1 (or 0), this strategy means that the agents are always locked into Room A (or B) no matter what happens.

According to these results, it is not hard to find that the minority game model with such a strategy structure is closely related to memory length m. And the historical information can only increase with 2^m.

In econophysics, minority game models have been widely used to simulate a special kind of complex adaptive systems, the stock markets [28,29,8]. Researchers always hope to generate similar stock market data through the minority game model. The stand or fall of this similarity often need to be tested to see whether model data have the same stylized facts as the real market data. Besides, the minority game can also be used to study competition problems about an unbiased distribution of resources [30–33].

Table 1
A model strategy table in the minority game with memory length $m = 2$.

Information	Choices
11	0
10	1
01	0
00	1

2.4. How to model agent-based systems

Agent-based modeling [34] plays an important role in the progress of complex systems researches. Differing from stochastic equations, agent-based models try to regenerate the evolution of a complex system via a bottom-up approach by means of simulating the behaviors of plentiful homogeneous or heterogeneous agents at the micro scale. There are two general ideas which can guide the design of a particular agent-based model. I shall discuss the two ideas in the following two subsections.

2.4.1. Abstracting real-world systems

The minority game is a famous agent-based model in the field of econophysics. As one can see from Section 2.3, it originates from the El Farol bar problem. There have been many kinds of modifications on the minority game. A particular one is to model on the stock market [8]. It can be seen that the minority game on stock market has many simplifications compared to the real market, such as the adoption of linear relation between excess demand and price change, the neglect of transaction costs, etc. Even under such simplifications, the minority game can still reproduce many statistical characteristics of the stock market successfully [8], which gives a clear illustration of the capabilities of agent-based models to reveal the endogenous mechanisms under financial markets. Hence, one important idea when trying to build an agent-based model is to abstract real-world systems. First, regulations in the associated real-world system should be written down one by one. Second, the importance of each regulation should be evaluated; key regulations should be introduced into the model, while trivial ones can be eliminated, in order to make the model simple and clear. Third, decision-making process for the virtual agents should be carefully designed to mimic the behaviors of the real-world human beings. Finally, one can complete the designs of an agent-based model by combining the simplified structure and a large number of interacting virtual agents. It can be seen that this idea guides our designs of all the models appearing in Sections 4–8.

One more thing which I want to discuss is the simplifications made in the agent-based modeling method. In real financial markets, it can be seen that the price of an asset is the reflection of every market participant's information-collecting and decision-making abilities. Moreover, there also exist communications among participants which may lead to the herding phenomenon. Clearly, if we want to include all these factors into our agent-based model, the model can become much more complicated. So we have to compromise by simplifying the model accordingly. But does our model lose its generality and reasonability instead? One may refer to the famous Ising model in statistical physics. In the Ising model, the time is discrete, which is clearly an unreal assumption in our real world. However, the model can regenerate many ferromagnetic phenomena very successfully. Hence, we can conclude that a proper simplification can wipe off trivial factors in the real-world system and make the model more powerful in the explanations of the real-world phenomena. But as we know, making a good simplification is not always a simple task.

Next, we are going to discuss the second approach for building agent-based models, that is, building models through physical models.

2.4.2. Borrowing physical models

Since many physical models have already been proven proper to explain related natural phenomena, it is worth of academic exploration to extend them to economic or social systems. Here, the Ising model is taken as an example for illustrating this idea.

The Ising model aims at studying the temperature dependence of magnetic susceptibility during the ferromagnetic phase transition. The model contains a large number of interacting spins which form a certain topological structure. It is usually assumed that (1) each spin only has two states, i.e., $\sigma_j = +\frac{1}{2}$ for the up direction and $\sigma_j = -\frac{1}{2}$ for the down direction; (2) The range of spin interactions is limited to the first neighborhood [35].

The fundamental physical picture of the ferromagnetic phase transition is that when increasing temperature, the dominant state (up or down) shown in the overall lattice changes through spin interactions. On one hand, the principle of least action requests that all spins are aligned in the same direction so that the spin interactions are at the lowest level. On the other hand, thermal motions tend to drive the directions of the spins to a random arrangement at which the system's entropy is the largest. The probabilities of spin states obey the Boltzmann distribution. As long as the system's temperature gets higher than the Curie temperature, the thermal motions among the spins become dominant so that a phase transition occurs from ferromagnetic to paramagnetic. For a spin, suppose its energy is E_+ for the up state (i.e., $\sigma_j = \left|+\frac{1}{2}\right\rangle$), and E_- for the down state ($\sigma_j = \left|-\frac{1}{2}\right\rangle$). When $E_- > E_+$, the probability for the spin to be in the up state $\left|+\frac{1}{2}\right\rangle$ in the next time step is

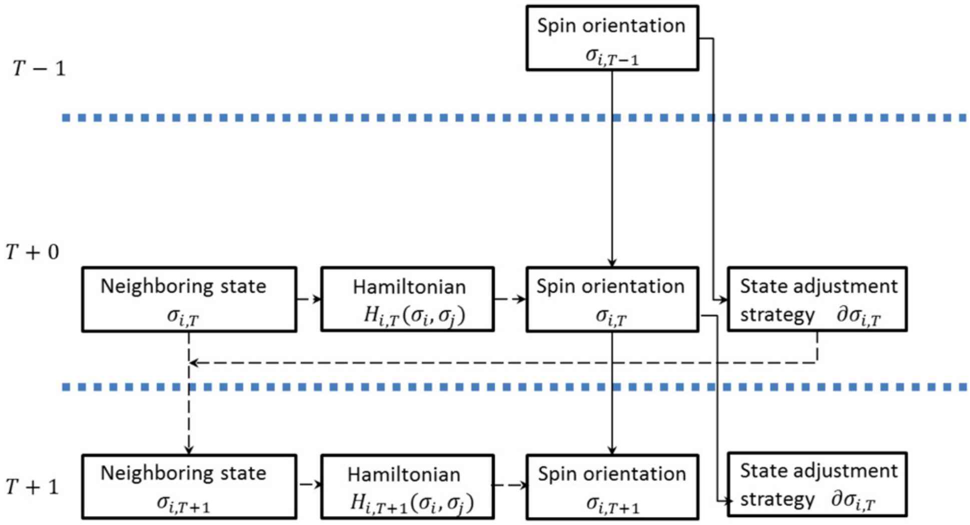

Fig. 4. Two-dimensional ferromagnetic phase transition in the Ising Model.
Source: Adapted from Ref. [20].

denoted as p_+, and in the opposite state $\left|-\frac{1}{2}\right\rangle$ with a probability of p_-. Then we can write,

$$p_+ = \frac{e^{-\frac{E_+}{kT}}}{e^{-\frac{E_+}{kT}} + e^{-\frac{E_-}{kT}}}, \qquad p_- = \frac{e^{-\frac{E_-}{kT}}}{e^{-\frac{E_+}{kT}} + e^{-\frac{E_-}{kT}}}.$$

It can be seen that when the temperature T gets closer to 0, $p_+ \to \frac{E_+}{E_+ + E_-}$, $p_- \to \frac{E_-}{E_+ + E_-}$, now all the spins tend to be arranged in the same direction. If T becomes much large, $p_+ \to p_- \to \frac{1}{2}$, which means that the directions of spins become totally random. That is, the system transfer from ferromagnetic to paramagnetic as T increases.

To have a further discussion of the Ising model, here we take the two-dimensional orthogonal cubic lattice for example. The Hamiltonian can be composed of two parts:

$$H(\sigma_j) = -\sum_i J_{i,j}\sigma_i\sigma_j - h_j\sigma_j.$$

Here, the first part $J_{i,j}\sigma_i\sigma_j$ represents interactions between spins (near-field or local effect), where spin i belongs to the first neighborhood of spin j. The second part $h_j\sigma_j$ is the Hamiltonian of σ_j in the external magnetic field h_j (global effect). According to the principle of least action, every spin tends to choose states that obey the Boltzmann distribution. Under this distribution, the probability of being the least energy state for $H(\sigma_j)$ is largest. The state probability is

$$P(\sigma_j) = \frac{e^{-\beta H}}{\sum_{\sigma_j} e^{-\beta H}},$$

where $\beta = k_B T$.

In the simulations, a spin in the lattice is selected randomly and the state of the spin is adjusted on the basis of thermodynamical laws. By repeating this procedure, the system can finally reach the equilibrium. At the equilibrium, one macroscopic property of the system simply equals the ensemble average of the associated microscopic property among all the spins. Note that $P(\sigma_j)$ is not independent of T, thus we can obtain the relation between one observation of the value of E and the temperature T or other parameters:

$$E(f) = \sum_j f(\sigma_j)P(\sigma_j).$$

A logical framework of the Ising model is shown in Fig. 4.

By comparing the similarities between ferromagnetic lattice and financial markets, one can establish an agent-based model for financial markets based on the logical framework of the Ising model (Fig. 5). The analogy between financial markets and ferromagnetic lattice is obvious.

Firstly, from the point of interactions, in the Ising model the spin states depend on the combined result of both global effect from the macro external magnetic field and local effect from the micro neighboring spins' states. And in the financial markets, investing strategies made by market participants also depend on the combined result of global information such as the price and trading volume of an asset, fundamental market information (like GDP or CPI), and local information

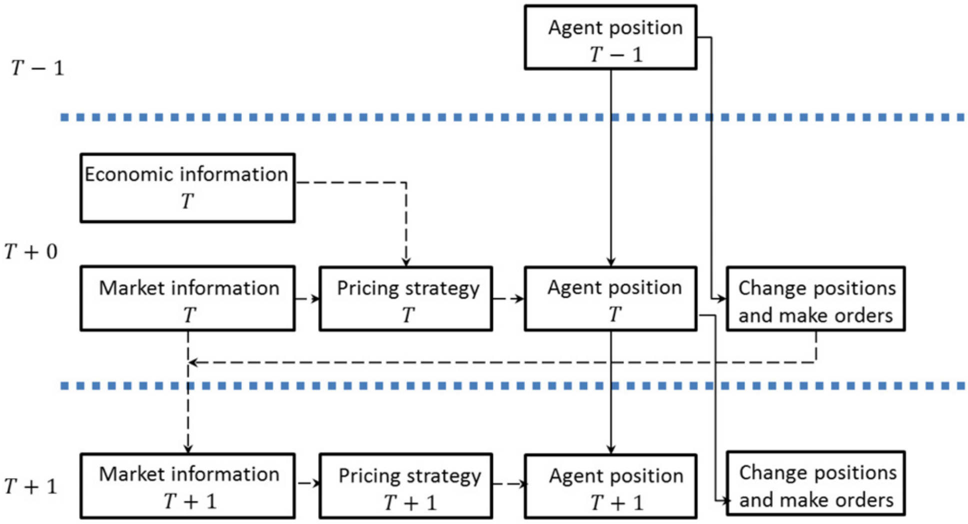

Fig. 5. Agent-based financial market model borrowing from the framework of the Ising Model in Fig. 4.
Source: Adapted from Ref. [20].

from the "neighboring" traders (here "neighboring" means the first neighborhood in the social network appearing in financial markets for a trader). Hence, we may write the associated "Hamiltonian" at one lattice grid for traders as $H(\sigma_j) = -\sum_i J_{i,j}\sigma_i\sigma_j - h_j\sigma_j$.

Secondly, from the point of "strategies", in the Ising model, spins always tend to change their states to find the best one which confirms to the least action and the maximum entropy principles; similarly, in the financial market model, agents should also be able to modify their investing strategies to find the best one which brings in maximum returns with minimum risks. Thirdly, from the point of feedback process, in the Ising model, the macro magnetic susceptibility can be obtained by summing up all the micro spin states; accordingly, in the financial market model, through the match of orders executed by market makers, price at each time is generated.

Thus, we can see that there exists a deep analogy between physical and economic models.

The financial market models based on the Ising model's framework have received a preliminary success [36,37]. But what strategies should be adopted by agents so that the real-world phenomena in financial markets can be regenerated? What kind of payoff function is able to reflect the investment demands properly? Do the time scales at which agents make decisions impact upon global information? All these questions are still waiting to be answered in the future.

3. Stylized facts

Inspired by H. E. Stanley and his coauthors' pioneering work [5,4], physicists began studying the statistical properties of financial markets with methods widely used in statistical physics. As a result, many universal rules, say, scaling laws (namely, power-law distributions in the eye of econophysicists) and clustering behaviors were empirically observed in the stock markets of different countries [5,4,38–48], or even in other fields including music [49] and linguistics [50]. Clearly, because it is illegal or immoral to control real markets, these researches lack controllability that, however, is very important to know the impact of a specific condition upon such universal rules. In this direction, S. P. Li and his coworkers [51] have conducted an experiment of political exchange for election outcome prediction, with a focus on the Taiwan general election in 2004. Future contracts for election outcome and corresponding options were created and traded among participants in a web-based market. The liquidation value was determined by the percentage of votes a candidate received on the day of election. It is a good prediction because participants do predict the vote percentage rather than just simply buy future contracts of the candidate they support. Finally the result was compared to that of the polls. Interestingly, when investigating the network topology of such an experimental futures exchange, researchers [52] showed that the network topology is hierarchical, disassortative and small-world with a power law exponent, 1.02 ± 0.09, in the degree distribution. They also showed power-law distributions of the net incomes and inter-transaction time intervals [52]. After identifying communities in the network as groups of the like-minded, they showed that the distribution of the community sizes is also distributed in the power law with an exponent, 1.19 ± 0.16 [52].

Inspired by their work [51,52], we believe that it is also possible to design stock markets in the laboratory, so that we can reveal the underlying mechanism of the universal rules.

In fact, economists have already done a lot of great work in laboratory human stock markets [53,26]. In 1990s, Friedman wrote an article to show his series of experiments; these experiments gave laboratory evidence of the efficiency of two different trading institutions [54]. Later, Porter and Smith designed a laboratory market with dividends, and also with several other extensions including short sells, limited price changing rules, associated future markets, etc.; they confirmed the

Fig. 6. An example of how our order book updates.
Source: Adapted from Ref. [57].

existence of price bubbles in this market and examined the extensions' roles in the bubbles [55]. Hirota et al. published an article based on a similar market structure; by studying the trading horizons, they suggested that the investors' short horizons and consequent difficulties of backward inductions are important contributors to the emergence of price bubbles [56]. These experiments offered good insights into the associated research problems. However, to our knowledge, all the experiments mentioned above have not convincingly reproduced the stylized facts (say, scaling laws) that have been revealed for real economic/financial markets by econophysicists. An important reason may be that in these experiments, time is divided into trading cycles, say 5 min. Hence the participators have to make decisions on the cycles [53,26,54–56]. In other words, discrete time steps make these laboratory markets deviated from real markets where trading time is naturally continuous. So, we attempt to overcome this problem by designing a continuous double-auction stock-trading market and carry out several human experiments in the laboratory [57]. As an initial work, the present artificial financial market can produce some stylized facts (clustering effects and scaling behaviors) that are in qualitative agreement with those of real markets. Also, it predicts some other scaling laws in human behavior dynamics that are difficult to achieve in real markets due to the difficulty in getting the data.

3.1. Market structure

3.1.1. Basic framework

Consider a market with N traders, indexed by i. Trading time is indicated by t. To simplify the problem, traders only decide how to manage their portfolios consisting of one stock and risk-free asset. The risk-free asset in our market is simply bank savings (cash). There are Q shares of stocks issued in the market. The price of the stock is determined by the traders' trading activities and it will be updated every time when a deal is made.

3.1.2. Double-auction order book

The double auction has been the most widely used system in equity markets for more than 140 years [58]. In our market, a computer-aided double-auction order book is introduced to help dealing with the traders' orders. Traders could have limit orders. Compared to a market order that only contains a desired amount of the stock to buy or to sell and will be executed on the current price, a limit order in addition has a request of specific limit of price. For example, a limit sell order with a bid price p and amount q means this trader is willing to sell q shares of the stock in any price no less than p. A limit buy order with an ask price p and amount q means this trader is willing to buy q shares of stock in any price no more than p. Traders could have unlimited numbers of orders, but neither borrowing nor short selling is allowed. Our order book works in the following ways:

1. At first the order book is empty;
2. When an order, for example, a buy order, is posted, the maximum amount of cash that may be needed is frozen;
3. The system will check if there are sell orders with lower prices. If there is no such order, store the order in the order book and the process is done; if there exist such orders, the system will pick out the ones with lowest prices, sort them by time to find the oldest one;
4. The system will exchange cash and stock between the owners of this order and the chosen orders in step 3; these cash and stock will be unfrozen and delivered to the account of the traders. If the order is fully digested, the process is done; if not, the rest part will be treated as a new order and repeat step 2 and 3.
5. When the order is aborted, the frozen cash or stock will be released to the traders' account. An example of our double-auction order system can be found in Fig. 6.

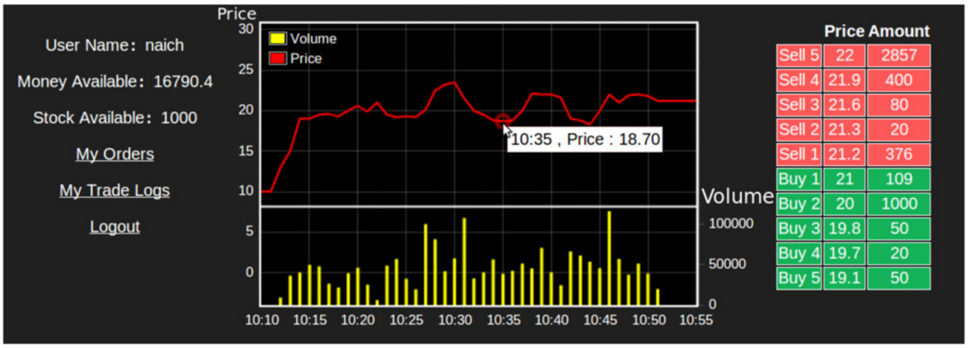

Fig. 7. A screenshot of the trading platform: the left part shows the trader's nickname, usable cash and stock (the number of shares); the middle chart demonstrates the stock price and trading volume as a function of time; the right table gives the five highest bid prices and five lowest ask prices. The middle chart refreshes per minute; when the mouse pointer (denoted by the white arrow) hovers above the stock price, it shows detailed information about time and the price of that time, say, "10:35, Price: 18.70" as shown in the chart.
Source: Adapted from Ref. [57].

3.1.3. Exogenous rewards

It is known that real stock markets are always full of various kinds of information, but such information has only two roles: tending to increase or decrease the stock price. Because the stock in our laboratory market has no underlying value, our solution is to add exogenous rewards to the system. For this purpose, we resort to dividends (that are used to potentially increase the stock price) and interests (that are utilized to potentially decrease the stock price) to give traders information about the macro environment and the stock. In detail, there will be stochastic rewards for holding stock or cash every a few minutes. The rewards for stock are a random amount of cash d directly added to traders' account; they are like the dividends in real markets. As a result, this may increase the stock price. The rewards for cash mean increasing the traders' cash by a random percent f, which represents the interest. So, this may decrease the stock price. The rewards also cover the stock and cash frozen in the order book. To let the traders have time to evaluate their strategies, all the rewards are forecasted with partial information by the coordinator 2 min before they are distributed. For example, if the coordinator is going to pay a dividend of "2 cash per share" at 10:00 am, he broadcasts to the traders that there will be a dividend of "1–3 cash per share" at 9:58 am.

3.2. Controlled experiments

3.2.1. Platform and subjects

We designed and conducted a series of computer-aided human experiments. The experiments were held in a big computer laboratory of Fudan University; each subject had a computer to work with. All the computers were linked to an internal local area network and we deployed a web server to handle all the requests. We recruited 63 subjects to act as traders, all of whom were students of Fudan University. Our trading platform provided the following information to the traders: 1-min close prices, 1-min trading volumes, five highest buy orders' prices and amounts, five lowest sell orders' prices and amounts, the trader's own cash and stock available, the trader's trading/ordering history and the trader's rewards-getting history. Details are shown in Fig. 7. According to our server's performance, the close price and volume were shown in a chart which was automatically updated every 1 min, the order book information was updated every 15 s, and the traders could look at their histories at any time during the experiments.

3.2.2. Experimental settings

Before the experiments, 10 min for trade training is arranged to help the subjects to get used to the trading interface and market rules. Then we had two rounds of experiments. Every round of experiment lasted for 30–40 min, but the traders did not know when the experiment would end, thus there would be less ending boundary effect. At the beginning of a new round, the stock price is set to 10. In the first round of experiment, all the traders started with 10 000 cash and 1000 shares of stock, while in the second round of experiment, the traders started with a random amount of cash and stock. In the second round, the traders' initial stock were randomly distributed between 200 and 1800, and to make the total amount of stock and cash comparable with the first round, every trader's initial cash is 10 times of his/her stock in number. In the first round of experiment, we initiated 63 000 shares of stock and 63 0000 cash; in the second round of experiment, we initiated 63 478 shares of stock and 634 780 cash.

3.2.3. Payoffs

Our experiments were carried out during the Econophysics course taught in Fudan University. The subjects were students enrolled in this course. 47 of the 63 subjects selected the course and 16 students were auditors. The performance of the students who selected the course took 10% of their final score of this course. They were required to trade at least 20 times

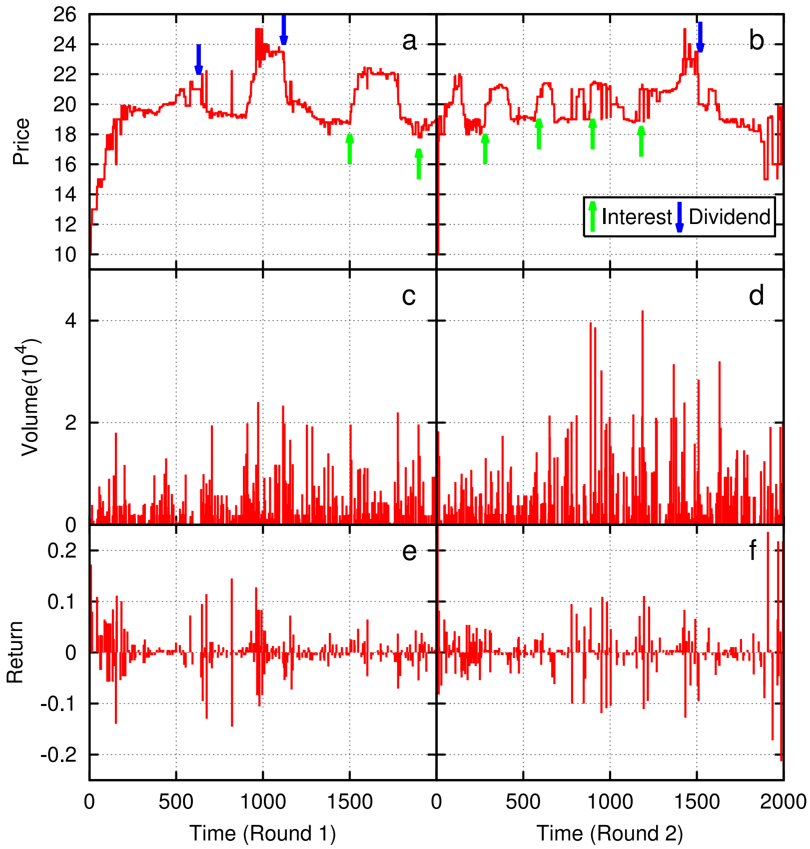

Fig. 8. Time series of (a–b) 1-min closing prices, (c–d) volumes and (e–f) returns for the (a, c, e) first and (b, d, f) second round of the laboratory human experiment. In (a) and (b), up arrows demonstrate the time when there is an interest (reward for cash), and down arrows demonstrate the time when there is an dividend (reward for stock). In (a), from left to right, the four arrows denote rewarding "2 cash per share", "2.4 cash per share", "10% of the cash", and "6% of the cash", respectively. In (b), from left to right, the arrows means rewarding "3%, 5%, 7%, and 9% of the cash" for the four up arrows, and "3.2 cash per share" for the down arrow. (c) and (d) show the trading activities lasting through the experiments. In (e) and (f), clustering behavior occurs.
Source: Adapted from Ref. [57].

to get a base score of 3.3%. And based on their final wealth ranking, they could get another 3.3%–6.7% score: based on their scores, top 10% students would get all of the 6.7% score, top 10%–30% would get 6.0% of the final score, . . ., and the last 10% would only get 3.3% score. Their final total scores were the calculated score rounded to the nearest whole number.

It is worth noting that the crucial role of markets is to let participants have chances to pursue profits. In different situations, "profits" could have different forms. For example, in real stock markets, investors pursue money ("profits") by exchanging stocks and money. In our laboratory market, 47 students who select the course pursue scores ("profits"), and the other 16 auditors voluntarily participated the experiments with an aim at learning how laboratory experiments are conducted for econophysics ("profits"). In this sense, our laboratory market can be equivalent to real stock markets, at least to some extent.

3.3. Results and discussion

3.3.1. Price, volume and return series

Here we show the data obtained from the human experiments (Fig. 8). The first is the 1-min close price series, $p(t)$. Here a 1-min close price is the last transaction price that occurs at the end of a certain minute: if there is no order execution in this minute, the close price will stay the same with the price of last minute. The (log) return $r(t)$ is defined as follows

$$r(t) = \ln p(t) - \ln p(t-1), \tag{1}$$

where trading time t is denoted by the count seconds from the start of the experiment. Fig. 8(a)–(b) give the price series of our experiments: during the first round of experiment, there are 2 interests (rewards for cash) and 2 dividends (rewards for stock), and during the second round of experiment, there are 5 dividends (rewards for stock) and 1 interest (rewards for cash). Because our rewards are forecasted 2 min in advance, there are notable price changes before the rewards' distribution. Specifically, before a reward for stock, the price goes up; before a reward for cash, the price goes down. This could be explained: when a signal of holding stock is sent to the traders, they tend to hold more units of stock. As a result, more

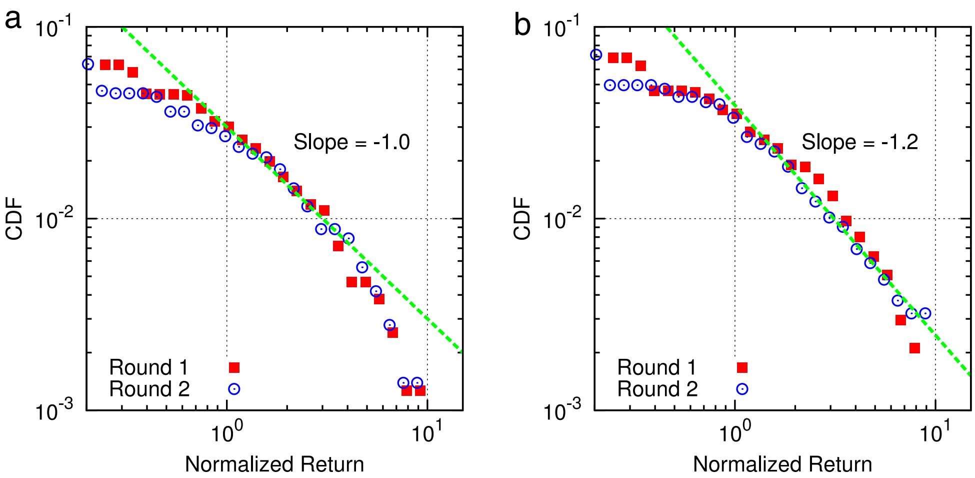

Fig. 9. Cumulative distribution functions (CDFs) of the normalized return: (a) the negative tails of the experiments and (b) the positive tails of the experiments. Symbols of squares and circles denote the results obtained from the first and second round of experiments, respectively. Here the negative tails in (a) denote the CDF (<0.064) of the absolute value of negative normalized returns, and the positive tails in (b) denote the CDF (<0.071) of the value of positive normalized returns. In (a) or (b), "Slope" denotes the slope of the corresponding green dashed line.
Source: Adapted from Ref. [57].

buy orders will come to the order book and pull up the price, and vice visa. However, if one buy the stock in a price much higher than the present price, he will gain less profit in this turn of reward. Thus the price will not go up infinitely. The same theory works for the case of rewards for cash. Because traders have different strategies and prediction of the future event, plus our forecast is not accurate, different traders have different responses to the news. This mechanism provides liquidity to our markets. Fig. 8(c)–(d) show the trading volume series. It is observed that there are trading constantly all the time.

Fig. 8(e)–(f) show the return series of the two rounds of experiments. It is easy to recognize that there are clusters in the return series; this is distinctly different from Gaussian random series. So we analyze the return series' statistical properties. In order to compare the data from the two rounds clearly, the normalized return, $g(t)$, is used,

$$g(t) = \frac{r(t) - \langle r \rangle}{\sigma(r)}. \tag{2}$$

Here $\langle \cdots \rangle$ denotes the average of time series $\cdots$, and $\sigma(\cdots)$ means the standard deviation of $\cdots$. We calculate the cumulative distribution function (CDF) of the first and second round, respectively. Since the returns distribute symmetrically around zero, we respectively calculate the positive returns and negative returns and put them in Fig. 9 for comparison. For the two rounds (Fig. 9), the negative and positive tails share almost the same CDF. Further, in the log–log plot all the four tails have a particular region that is approximated by a straight line. Clearly this behavior is the evidence of scaling, and meets the statistical analysis of many real stock markets [39].

The autocorrelation function is another important feature of the return series [40]. If $x(t)$ is a time series, the autocorrelation function, $C(\Delta T)$, is defined as

$$C(\Delta T) = \frac{\langle (X_{-\Delta T} - \langle X_{-\Delta T} \rangle)(X_{\Delta T} - \langle X_{\Delta T} \rangle) \rangle}{\sqrt{\sigma(X_{-\Delta T})\sigma(X_{\Delta T})}}, \tag{3}$$

where $X_{-\Delta T}$ is the series with the last elements removed and $X_{\Delta T}$ with the first elements removed. $\langle X \rangle$ denotes the average of X and $\sigma(X)$ the standard deviation. Our calculations of $g(t)$ confirm the existence of short negative correlation (less than 20 s or so) on both rounds of experiments [as indicated by the green dashed line in Fig. 10(a)], which shows our laboratory market is similar to the real developed stock markets [59]. The short time correlation also fits the refreshing time of our order book information (15 s). We also calculate the autocorrelation of absolute normalized return, $|g(t)|$, and find that the correlation lasts much longer than 20 s [see Fig. 10(b)]. This result confirms the volatility clustering behavior in our market, and echoes with many other articles, for example, see Refs. [43,59]. In addition, if we compare the two rounds, we can conclude that in the present market, the initial wealth distribution has little influence on the statistical properties of return.

3.3.2. Human behavior dynamics

Our market experiments also give us an opportunity to study human behavior dynamics. In 2005, Barabási et al. analyzed the letters of Darwin and Einstein; they found that both Darwin's/Einstein's patterns of correspondence and today's electronic exchanges follow the same scaling laws [60]. And they use an agent-based model to explain the origin of this scaling [61]. Here we turn our eyes to the waiting time of traders' actions. We define two kinds of waiting time, the stock waiting time and the trader waiting time. The stock waiting time describes the gaps between which two different orders

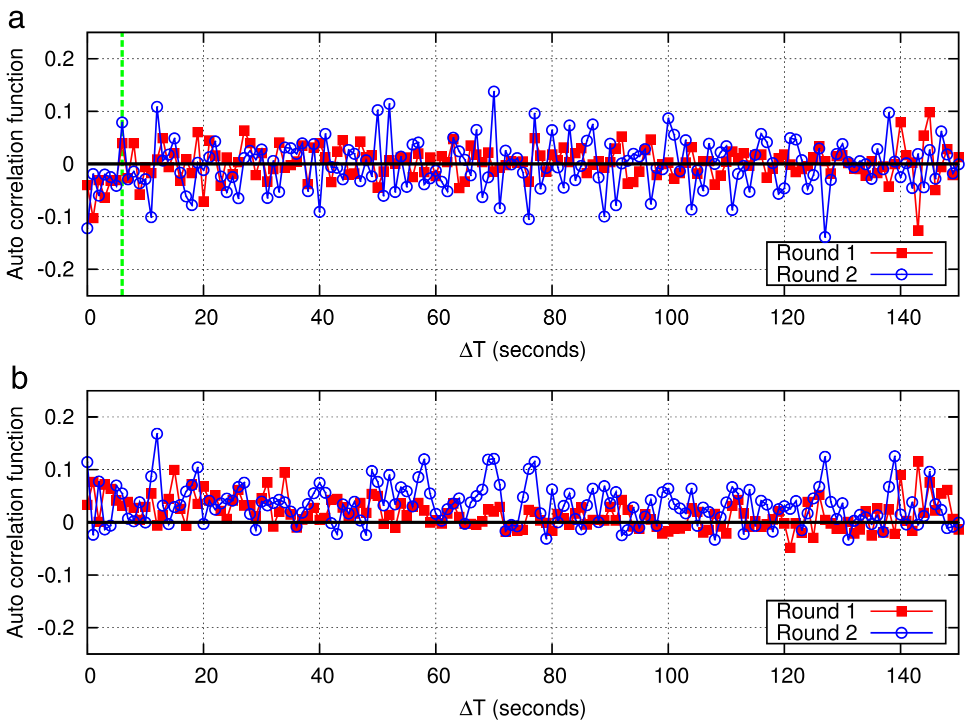

Fig. 10. Autocorrelation function of (a) normalized return series, $g(t)$, and (b) absolute normalized return series, $|g(t)|$, for Round 1 and Round 2. In (a), the green dashed line indicates the transition point. Details can be found in the text.
Source: Adapted from Ref. [57].

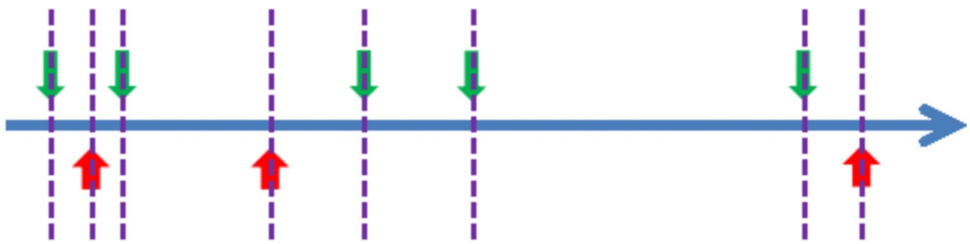

Fig. 11. Demonstration of the definition of waiting time. The axis represents the time. Assume our market has only two traders, the first trader's orders are marked with down arrows, and the second trader's orders are marked with up arrows. The stock waiting time is shown by the gaps between each pair of nearby slash lines, and the trader waiting time is indicated by the gaps between each pair of nearby down arrows and the gaps between each pair of nearby up arrows.
Source: Adapted from Ref. [57].

are posted, see Fig. 11. We put all the orders from all the traders together, sort them by time, and calculate the time gaps between two successive orders. While the stock waiting time is to focus on the collective behavior of a group, by defining the trader waiting time we try focusing on the decision-making processes of individuals. For the trader waiting time, we put orders from different traders in different piles, sort them respectively, and then calculates the gaps. This is because for any particular trader she/he only has tens of orders, this is insufficient for statistical analysis. Instead, if we are looking into the rules that work across the crowds, we could put all the gaps together, thus we get thousands of data. This method has been used in the literature on human behavior dynamics [62,63].

The probability density function (PDF) is calculated; see Fig. 12. Fig. 12(a) demonstrates the PDF of stock waiting time in a log plot graph. The data points obviously locate in a straight line, which means the stock waiting time obey an exponential distribution [61]. This is because the traders have little interactions when submitting orders, their actions can be seen as independently decisions and overall exhibit a random-like behavior. However, in Fig. 12(b), we could find the trader waiting time is quite different. The PDF in a log–log plot forms a straight line for the gaps that are shorter than 100 s, and the tail of the PDF drops blow the line when the gap is longer than 100 s. In the previous literature, Barabási showed that the power-law distribution of waiting time may come from a priority queuing system [61]. In our market, when traders make decisions on whether he/she should submit an order, there is no obvious use of a task queue. So the origin of power law in trader waiting time may contain some other mechanism. The turning point from which the PDF's tail drops from the straight line

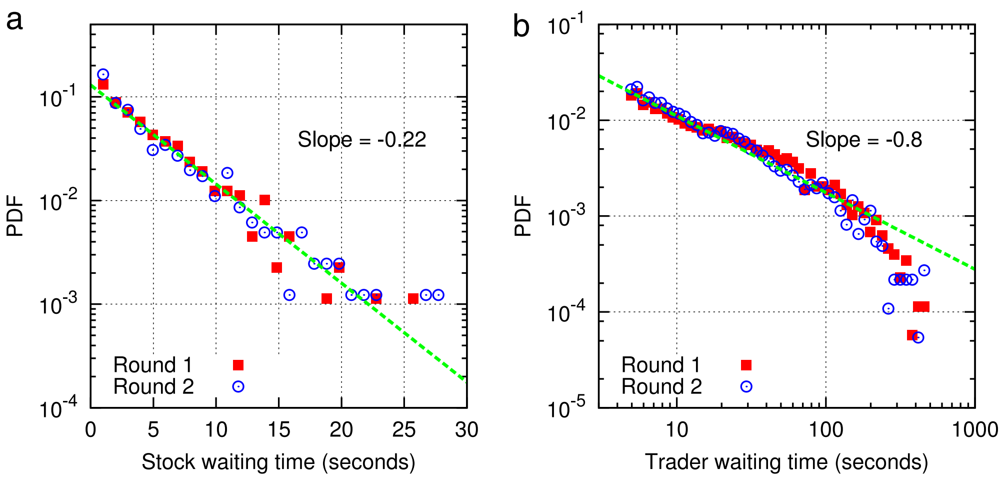

Fig. 12. Probability density functions (PDFs) of waiting times: (a) the stock waiting time and (b) the trader waiting time. Symbols of squares and circles denote the first and second round of experiments, respectively. In (a) or (b), "Slope" denotes the slope of the corresponding green dashed line.
Source: Adapted from Ref. [57].

fits our rewarding time gaps in magnitude. We suspect that the tail of PDF may present the effect of our exogenous rewards. Those news breaks the traders original decision making process. For example, if there is no news in the market, a trader might trade every 20 min; however, if there are periodic news every few minutes, he/she is likely to respond according to the news. Therefore, the gap of 20 min will no longer exist. In a word, we believe the lack of long waiting time causes the fall of the tail in Fig. 12(b).

3.4. Concluding remarks

In contrast to the existing laboratory markets where trading time was set to be discrete [21,64,65,53–56,58], we have designed a double-auction stock-trading market where trading time is continuous. We have run two experiments in laboratory with human subjects and found that the initial outputs of the market fit some existing stylized facts (clustering effects and scaling behaviors). Besides, we have analyzed the orders and discovered some scaling laws in human behavior dynamics. Our laboratory market still has some weak points. For example, the traders of our experiments are all university students, and there might be differences if we choose different groups of traders beyond university students. However, as a model market, it is easy for us to change different control parameters or add more extensions. Thus this market is expected to produce more results in future researches, such as, on either the effect of leverage [66,67] or the birth of bubbles. The market and its output might also help modelers to mimic human behaviors in a more precise and realistic way. Section 3 shows a way to study real stock markets by conducting controlled experiments on laboratory stock markets producing high frequency data.

4. Herd behavior

Section 3 has discussed some fluctuation phenomena in laboratory stock markets, namely, overall fluctuations and extremely big fluctuations. In reality, the latter may lead to crises. It seems to be a common knowledge that crises always originate from herd behavior of human beings (see for example Ref. [68]). Clearly this common knowledge displays the ruinous role of herd behavior. The aim of Section 4 is to show the different roles of herd behavior.

Most of the social, ecological, and biological systems that involve a large number of interacting agents can be seen as complex adaptive systems (CASs), since they are characterized by a high degree of adaptive capacities to the changing environment. Their dynamics and collective behaviors have attracted much attention among physical scientists [69–72,50]. In order to survive, self-serving agents in these CASs must compete against others for limited resources with biased or unbiased distribution by conducting strategic behaviors. This can globally result in balanced or unbalanced resource allocation. Examples of such phenomena evolves many species like human beings. For instance, drivers select different traffic routes, people bet on horse racing with odds, and so on. In general, the allocation of the resources in a CAS could reach a balanced state due to the preferences and decision making ability of agents, as revealed by investigating a biasedly distributed resource allocation problem [21]. In practice, however, it will sometimes fail to reach the balanced state. For this, one important reason is due to the formation of a herd. In fact, herding extensively exists in collective behaviors of many species in CASs, including human beings. Though human decisions are basically made according to individual thinking, people tend to pay heed to what others are doing, emulate successful persons, or those of higher status, and thus follow the current trend. For example, young girls often copy the clothing style of some famous stars named as trendsetters in the fashion world. Similarly, researchers would rather choose to work on a topic that is currently hot in the scientific society. As a result, large numbers of people may act in concert, and this unplanned formation of crowds is called herd behavior [73]. Locally, for an individual agent, herd behavior may suggest either irrationality [74,75] or rationality [76–78], with an implication that herding can ruin the balance of the whole resource-allocation system by causing excess volatility. Accordingly, herd behavior is commonly seen as a tailor-made cause for explaining bubbles and crashes in a CAS with the existence of extremely high volatility. But is this ''common sense'' always right? Based on results of this study, we argue that herd behavior should not be labeled like the killer all the time. Here we focus on the effect of herding on the whole CAS for resource allocation, because it is most important for as many agents (involving human beings) as possible to survive in various kinds of CASs like social, ecological or biological systems. As a result, we shall not study or consider the details on how to reach a herd through contagion and/or imitating because our results are not dependent on the process of herding formation.

4.1. Controlled experiments

To model the realistic huge systems of resource allocation including the effects of herding, we design and conduct a series of computer-aided human experiments, on the basis of the resource-allocation system [21,7,79], in order to study the necessary conditions for a CAS to reach the ideal balanced state. Using this kind of experimental settings will allow us to investigate the herd behavior in a well regulated abstract system for resource allocation, which reflects the fundamental characteristics of many CASs [31,80,81]. Human subjects of the resource-allocation experiment were students recruited from several departments of Fudan University. Before the start of experiments, a leaflet (as shown in Part I of the Appendix of Ref. [64]) was provided which explains configurations of the experiment and actions of the subjects. There are two rooms (Room 1 and Room 2) and the amounts of resource in these two rooms are M_1 and M_2 ($\leqslant M_1$), respectively. As the experiment evolves, M_1 and M_2 are kept fixed and unknown to all the subjects. For each experiment round, each subject has to choose one of the two rooms to enter. Those who go into the same room should share alike the virtual resource (M_1 or M_2) in it. Apart from human subjects, there are also imitating agents joining the experiment. All the imitating agents are generated by a computer program, since their decisions are simply made by mimicking human subjects' behaviors. In particular, each imitating agent will randomly select a new group (of size 5) of human subjects at every experiment round, and then follow the choice of the best subject (who has the highest score) in the group for the next round. In each round of the experiment, the number of human subjects and imitating agents in Room 1 is denoted as N_1 and the number in Room 2 as N_2. Therefore the total number of human subjects and imitating agents can be counted as $N = N_1 + N_2$. The human subjects or imitating agents who earned more than the global average $(M_1+M_2)/N$ are regarded as winners of the round, and the room which the winners had entered as the winning room. The total number of human subjects or imitating agents can also be expressed as $N = N_n + N_m$. Here N_n is the total number of human subjects who make decisions by their own, and N_m is the total number of imitating agents who do not have their own ideas. The ratio between imitating agents and human subjects is defined as $\beta = N_m/N_n$. More details about the experiment can be found in Part II of the Appendix of Ref. [64].

The resource-allocation experiments are conducted repeatedly with different values of M_1/M_2 and β. The modeled system is designed as an open system in which the number of human subjects N_n is fixed while the number of imitating agents N_m is increased in an implicit manner. As shown in the previous study [21], the heterogeneity of preferences is an indispensable factor for the whole system to reach the balanced state. Hence the preferences of human subjects need to be checked under the influence of imitating agents. For a human subject in the experiment, his/her preference is evaluated as the average rate that he/she chooses to enter Room 1. Preferences of the 44 subjects are plotted in Fig. 13 with different M_1/M_2 s and/or β s. Fig. 13(a) shows the preferences of human subjects when $M_1/M_2 = 1$ and the imitating agents are absent. Distinctions among the preferences of human subjects can be easily identified. For example, the 4th subject is strongly partial to entering Room 2 while the 6th subject prefers Room 1 much more. It can be found in Fig. 13(b,c), that the human subjects still have diverse preferences even when M_1 becomes much larger than M_2. In addition, the heterogeneity of preferences remains even for the cases in which $N_m(= N_n/2)$ imitating agents are involved; see Fig. 13(g–i). Despite of this heterogeneity, the average of subjects' preferences changes along with M_1/M_2. In other words, human subjects have the ability to adapt themselves to fit the environment.

Comparisons of the distributions of human subjects' preferences, as the resource distribution M_1/M_2 is varied and/or the imitating agents are involved, are shown in Fig. 13(d–f,j–l). From Fig. 13(d,e), one can find that when M_1/M_2 is not so biased, human subjects alone can do the analysis of the system so well that they can make the whole system reach the balanced state. Note that the preference distribution has a peak at 0.5 in Fig. 13(d) and the subjects' preferences are mainly distributed around 0.75 in Fig. 13(e). Both of the two observations can be deduced from the resource distribution, $M_1/M_2 = 1$ and $M_1/M_2 = 3$. When the imitating agents are involved, however, the two preference distributions have some changes in Fig. 13(j,k). In particular, the peak almost disappears in Fig. 13(j) and the mean value of subjects' preference deviates from the resource distribution bias in Fig. 13(k). A possible reason for these changes can be inferred as that human subjects may get confused by the behavior of imitating agents. Hence in this case the herd (which is formed by imitating agents) indeed disturbs the system and weakens the analyzing ability of human subjects. Things are different if M_1/M_2 gets even larger, as shown in Fig. 13(f,l). Here the involvement of imitating agents does not bring much change to the preference distribution of human subjects. One may say that, in this case, herd behavior has no harmful effect on the analyzing ability of the human subjects. Finally, it is interesting to note from the same figure, that a minority of human subjects with preference to Room 2 can stay alive even in a highly biased system ($M_1/M_2 \gg 1$) when the imitating agents exist.

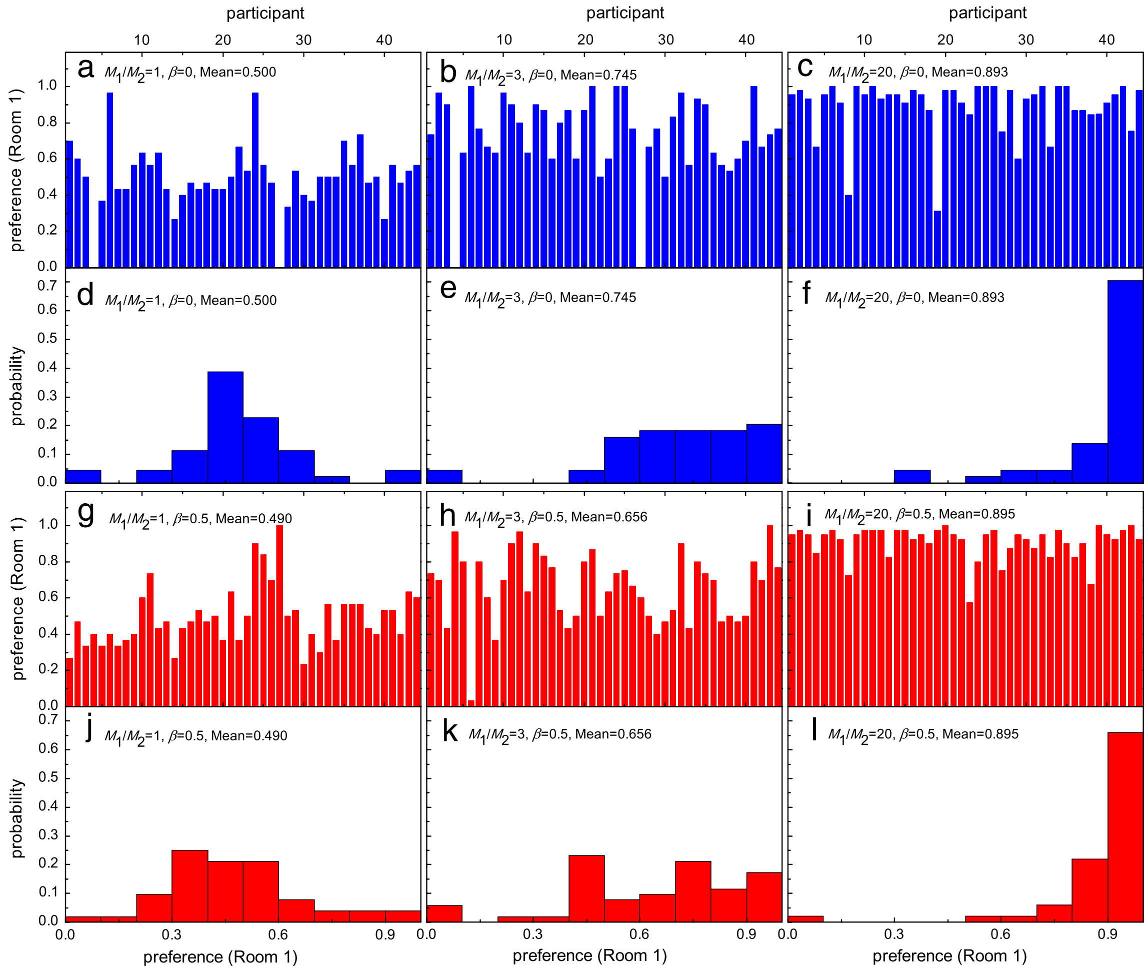

Fig. 13. Data obtained from the human experiment. (a–c,g–i) Preferences of the 44 subjects in sequence to Room 1 for the cases (a–c) without and (g–i) with imitating agents, $\beta = rm(a - -c)0$ and (g–i) 0.5, for the resource distributions $M_1/M_2 =$ (a, g) 1, (b,h) 3, and (c,i) 20. Here, "Mean" denotes the average value of the preferences of the 44 subjects. (d–f,j–l) Distribution of the 44 subjects' preferences.
Source: Adapted from Ref. [64].

To evaluate the performance of the whole system, we have calculated efficiency (which, herein, only describes the degree of balance of resource allocation), stability, and predictability of the resource-allocation system. The efficiency of the whole system can be defined as $e = |\langle N_1\rangle/\langle N_2\rangle - M_1/M_2|/(M_1/M_2)$. A smaller e means a higher efficiency in the allocation of resources. The stability of the resource-allocation system can be described as $\sigma^2/N \equiv \frac{1}{2N}\sum_{i=1}^{2}\langle(N_i - \tilde{N}_i)^2\rangle$, where ⟨A⟩ denotes the average of time series A. This definition describes the fluctuation (volatility) in the room population away from the balanced state, where the optimal room populations $\tilde{N}_i = M_iN/\sum M_i$ can be realized. The predictability of the system is measured by the "uniformity" of the winning rates in different rooms. The winning rate in Room 1 is denoted as w_1. It is obvious that if w_1 is close to 0.5, choices of the two rooms are symmetrical and the system is unpredictable. If the winning rate were too biased, smart subjects should be able to predict the next winning room in the experiment. As shown in Fig. 14, when M_1/M_2 is small ($M_1/M_2 = 1$ or 3), adding some imitating agents will lower the efficiency and cause large fluctuations. On the other hand, when M_1/M_2 gets even larger ($M_1/M_2 = 20$), the formation of herd can improve the efficiency, the stability, and the unpredictability of the resource-allocation system.

4.2. Agent-based modeling

An agent-based model is developed in order to fully understand the preceding experimental results. Consider a situation where N agents repeatedly join a resource-allocation system. Among these agents, there are N_n normal agents (which correspond to human subjects in the preceding experiments) and N_m imitating agents, so that the total number of agents can be calculated as $N = N_n + N_m$. To play in the resource-allocation system, each normal agent will take S strategies from the full strategy space and compose a strategy book. A strategy for the resource-allocation experiment is typically a choice table which consists of two columns. The left column is for the P possible situations, and the right column is filled with bits of 0 or

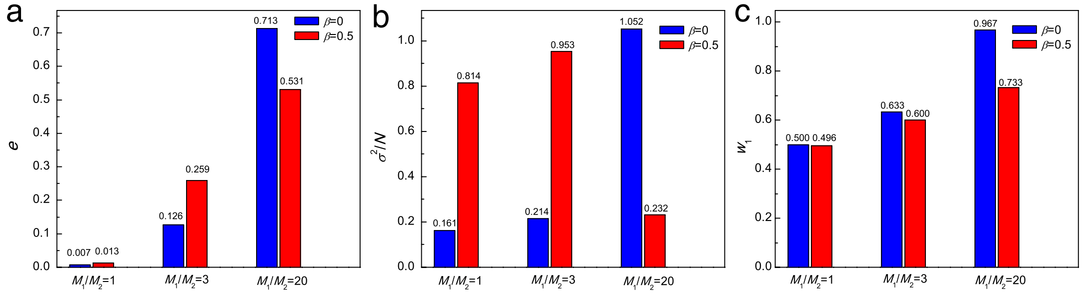

Fig. 14. Experimental results for (a) efficiency e, (b) stability σ^2/N, and (c) predictability w_1 of the modeled resource-allocation system, with human subjects $N_n = 50$. $\beta = 0$ and 0.5 correspond to imitating agents $N_m = 0$ and 25, respectively. Each experiment lasts for 30 rounds.
Source: Adapted from Ref. [64].

1. Bit 1 is linked to the choice for the entrance of Room 1, while bit 0 to that of Room 2. In the strategy book of a normal agent, strategies differ from each other in the preference, which is defined as an integer $L\,(0 \le L \le P)$. To model the heterogeneity of preference, let the normal agent pick up a preference number L first. Then each element of the strategy's right column is filled in by 1 with the probability L/P, and by 0 with the probability $(P - L)/P$ (more detailed explanations can be found in Part III of the Appendix of Ref. [64]). The process will be repeated S times, each time with a randomly chosen L for each normal agent to complete the construction of its strategy book. From the start of the resource-allocation experiment, each normal agent will score all the strategies in its strategy book so as to evaluate how successful they are to predict the winning room. Following the hitherto best performing strategy in their strategy books, normal agents are enabled to make decision to enter one of the two rooms, once the current situation is randomly given.[1] Imitating agents in the model behave in a different way during the process of decision making. Before each round of the play starts, each imitating agent will randomly select a group of k $(1 \le k \le N_n)$ normal agents.[2] Within this group, the imitating agent will find the normal agent who has the best performance so far and imitate its behavior in the following experiment round. It is assumed that the imitating agents know neither the historical record of the winning room nor the details of strategy books of other group members. The only information for them to access is the performance of the normal agents, that is, the virtual money that these normal agents have earned from the beginning of the experiment. If the number of imitating agents N_m kept increasing, there would be more and more positive correlations among agents' decisions, which would trigger the formation of a herd in the system.

4.3. Simulation results

Agent-based simulations are carried out in an open system condition, in reference to the experiments. (Please refer to Part IV of the Appendix of Ref. [64] to see the results for a closed system.) Following the analysis of experimental results, we first investigate the simulation results for the preferences of normal agents. Clearly, Fig. 15 shows distributions of the preferences similar to those shown in Fig. 13. The qualitative agreement indicates that our agent-based modeling has taken into account the heterogeneity of preferences with a reasonable modeling of the decision making process for the human subjects. (We had also investigated the preferences of normal agents in an alternative way by analyzing the Shannon information entropy; see Part V of the Appendix of Ref. [64].) Next, efficiency, stability, and predictability of the whole modeled system are calculated according to the definitions made in the experimental study. The change of system behavior along with the variation of the resource ratio M_1/M_2 is shown in Fig. 16. Differently colored symbols in the figure represent results obtained under different values of β. As shown in Fig. 16(a), when the resource distribution is comparable ($M_1/M_2 \approx 1$), the averaged population ratio $\langle N_1\rangle/\langle N_2\rangle$ can always be in concert with M_1/M_2 no matter imitating agents are involved or not. On the other hand, as the resource distribution gets more and more biased (M_1/M_2 increases), surprisingly the whole system tends to reach the balanced state only if more imitating agents (larger β) join the system. Fig. 16(b) shows the change of efficiency of the resource-allocation system. The tendency is that when the resource ratio gets more biased, a larger size of herd is needed to realize a higher efficiency of the resource distribution. From both the sub-figures, the so-called "M_1/M_2 phase transition" [21], where M_1/M_2 plays the role of control parameter, can also be identified. As shown in Fig. 16(c), the increase of the number of imitating agents will cause larger fluctuations in the low M_1/M_2 region. However, as M_1/M_2 increases, more imitating agents can yield higher stability of the resource-allocation systems. Comparing system behaviors for the cases of $\beta = 0$ and $\beta \neq 0$, the M_1/M_2 phase transition also indicates the change of role for the herd behavior, namely, from a ruinous herd into a helpful herd. It is clear that the critical point of the M_1/M_2 phase transitions get larger when the number of imitating agents increases. Denoted as $(M_1/M_2)_c$ hereafter, the critical point refers to the M_1/M_2 value where the

[1] Here the situation is not the history of winning rooms. Broadly speaking, it can be explained as a mixture of endogenous and exogenous system information. Results obtained with the real history bit-strings have no essential difference with the current study, though the use of random information makes the theoretical analysis easier.

[2] This corresponds to the case of primary imitators. In fact, in the real system, there might exist multi-level imitations where some imitators can copy other imitators' behavior. Similar conclusions could be achieved.

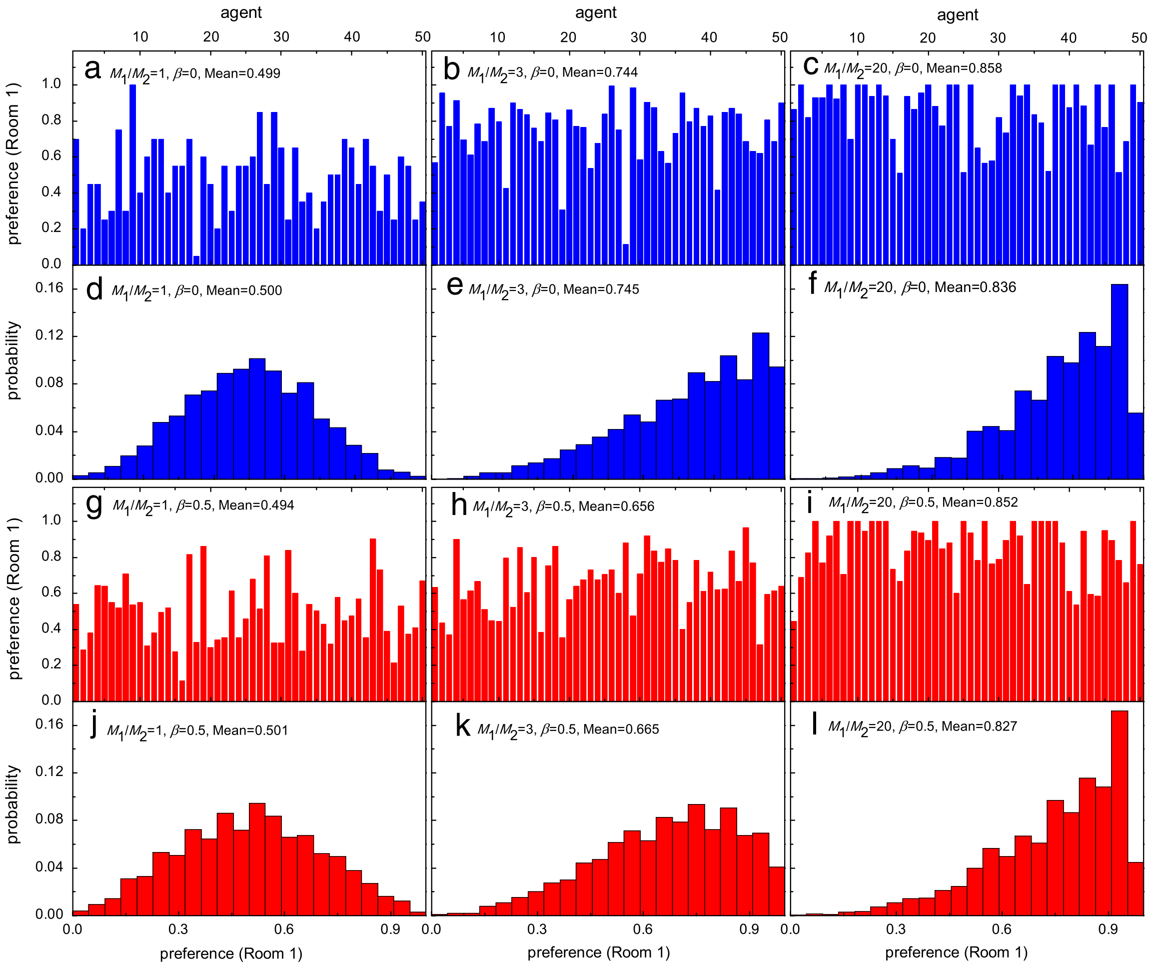

Fig. 15. Simulation results obtained from the agent-based simulations. (a–c,g–i) Preferences of the 50 normal agents to Room 1 for the cases (a–c) without ($\beta = 0$) and (g–i) with ($\beta = 0.5$) imitating agents, and for the resource distributions $M_1/M_2 =$ (a, g) 1, (b,h) 3, and (c,i) 20. We have run the simulations for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). (a–c,g–i) are typical results of one of the 200 runs. In (a–c,g–i), "Mean" denotes the mean value of the preferences of the 50 normal agents. (d–f,j–l) Distribution of the 50 normal agents' preferences. Note that (d–f,j–l) are obtained from the average over the 200 runs, and also the "Mean" in (d–f,j–l) denotes this average. Simulation parameters: $S = 4$, $P = 16$, and $N_n = 50$.
Source: Adapted from Ref. [64].

minimum is achieved. This definition together with the mechanism for the increase of $(M_1/M_2)_c$ will be further discussed in the theoretical analysis of the model. Finally, the effect of herd behavior on the predictability of the resource-allocation system is shown in Fig. 16(d). When more imitating agents are introduced to the system for large M_1/M_2, the prediction of the next winning room becomes more difficult as winning rates for the two rooms are more symmetric. Notice that the system behavior under various conditions found herein by the agent-based simulations echoes with the observations in the experiment.

We summarize the simulation results here and make some more comments to emphasize the significance of findings in our study. The performance of the resource-allocation system consisting of normal agents or human subjects with the full decision-making ability is, in some cases, inferior to those including imitating agents (who form the herd). This might seem questionable at first sight. In particular, it may be argued that the failure to reach the balanced resource allocation for large M_1/M_2 when $\beta = 0$ is only due to the relatively small population of the normal agents. However, it has been proved in the theoretical analysis (see the equation for the population in the next section) and the agent-based simulation of resource-allocation systems [21] that the total number of agents is indeed not a key factor. When the resource distribution is not biased so much, the normal agents can play pretty well so that the resource-allocation system behaves in a healthy manner (efficient or balanced, stable and unpredictable). In such kind of situations, adding imitating agents will only bring about a "crowded system" in which larger fluctuations (volatility) turn up. In this respect, our study shares some common features with the Binary-Agent-Resource model [82,83]. In particular, the "crowd effect" has been observed in these models and the inclusion of imitating agents in our model can be explained as a special kind of networking effects. Only if the resource distribution becomes so biased that most of the normal agents cannot completely solve the decision-making problem by

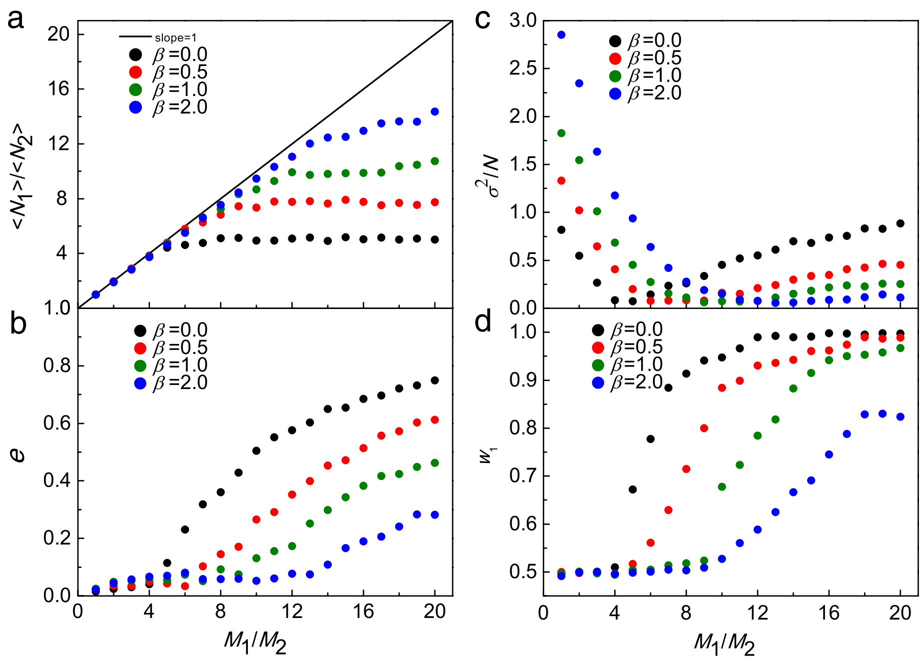

Fig. 16. (a) $\langle N_1\rangle/\langle N_2\rangle$, (b) e, (c) σ^2/N, and (d) w_1 as a function of M_1/M_2, for an open system in the agent-based simulations. Parameters: $N_n = 50$, $S = 4$, $P = 16$, $k = 5$, and $\beta = 0$, 0.5, 1.0, and 2.0. For each parameter set, simulations are run for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). In (a), "slope = 1" denotes the straight line with slope being 1.
Source: Adapted from Ref. [64].

referencing their strategy books, adding the imitating agents could become a helpful factor in consuming the remained arbitrage opportunities in the system. This explains the reason why the herd behavior in the resource-allocation system can effectively help the system to realize the balanced state and reduce instability and predictability in the mean time.

4.4. Theoretical analysis

To further understand the underlying mechanism for these phenomena, we also conduct theoretical analysis by deriving the critical points $(M_1/M_2)_c$ for the M_1/M_2 phase transition identified in the agent-based simulations. (For the details of derivation, please refer to Part III of the Appendix of Ref. [64].) As a result of the theoretical analysis, the maximum of population ratio in Room 1 $\langle R_1(=N_1/N)\rangle_{\max}$ can be obtained under the condition $M_1 \geq M_2$. It reads as the following (the meaning of the symbols can also be found in Part III of the Appendix of Ref. [64]),

$$\langle R_1\rangle_{\max} = 1 - \frac{1}{(\beta+1)P}\sum_{\tilde{L}=1}^{P}\left[\left(\frac{\tilde{L}}{P+1}\right)^{s} + \beta\left(\frac{\tilde{L}}{P+1}\right)^{ks}\right],$$

where $\tilde{L}$ stands for the preference of a normal agent's strategy. If $\langle R_1\rangle_{\max} = M_1/(M_1+M_2)$, the system can fluctuate around the balanced state. Otherwise, the system can never reach the balanced state. Then some insightful comments can be added:

- The state of the resource-allocation system depends only on M_1/M_2, β, k, P and S. It has no concern with N_n or N_m.
- An optimized value of β may be calculated by setting $\langle R_1\rangle_{\max} = M_1/(M_1+M_2)$, which could make the system most stable. After substituting this expression into the equation for $\langle R_1\rangle_{max}$, we can obtain numerical solutions for the critical points $(M_1/M_2)_c$ of the phase transitions. Fig. 17 shows a good agreement between the simulation results and those of theoretical derivation for the critical points.
- It is easy to prove that $\partial\langle R_1\rangle_{\max}/\partial\beta > 0$, which means that β and $\langle R_1\rangle_{\max}$ are positively related. When $\beta \to \infty$, the population ratio will converge to $\langle R_1\rangle_{\max} \to 1 - \frac{1}{P}\sum_{\tilde{L}=1}^{P}(\frac{\tilde{L}}{P+1})^{ks}$. At this limit, the model suggested here will be equivalent to the original resource-allocation model without the imitating agents [83], except that in this case, each agent would occupy kS (instead of S) strategies.

4.5. Concluding remarks

We have revealed that, if the bias between the two resources M_1/M_2 were large and is unknown to the subjects/agents, a herd of a typical size could help the overall system to reach the optimal state, namely, the state with a minimal fluctuation,

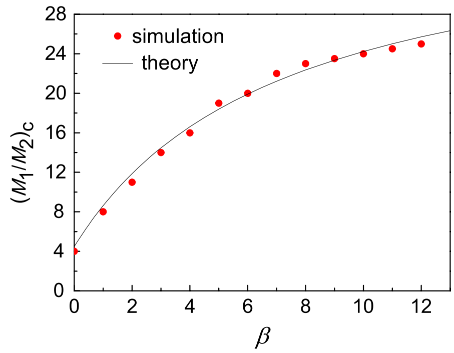

Fig. 17. Critical points of the M_1/M_2 phase transition, $(M_1/M_2)_c$, varying with different population ratios β: simulation results (symbols) versus theoretical results (line). The simulation results are obtained from the data in Fig. 16(a,c).
Source: Adapted from Ref. [64].

a high efficiency, and a relatively low predictability. The corresponding ratio between the two resources also works as the critical point of a class of M_1/M_2 phase transition. The phase transition can be used to discover the role change of herd behavior, namely from a ruinous herd to a helpful herd as the resources distribution gets more and more biased. The main reason for this generalization could be understood as follows. When a large bias exists in the distribution of resource, the richer room will offer more arbitrage opportunities so that it deserves to be chosen without too much deliberation. Since imitating agents learn from the local best human subject or normal agent, the herd formed by these agents will certainly be more oriented to the richer room. To balance a highly biased resource distribution, in fact, it correspondingly needs a suitable number of participants who have a highly biased orientation in their choices. But every coin has two sides. Normal agents will be confused if too many imitating agents are involved. Because in that case, they have to estimate not only the unknown system but also the behavior of the herd. The effect of herd behavior would become negative again under these situations. We emphasize that these arguments are quite general. In particular they are independent of the process of herding. In Part VI of the Appendix of Ref. [64], results of a different agent-based model, in which imitating agents follow the majority of the linked group, rather than the best normal agent, are shown. Similar results are achieved indeed.

Section 4 is also expected to be important to some fields, ranging from management and social science, to ecology and evolution, and to physics. In management and social science, administrators should not only conduct risk management after the formation of herd, but also need to consider system environment and timing to see whether the herd is globally helpful or not. In ecology and evolution, it is not only necessary to study the mechanism of herd formation as usual, but also to pay more attention to the effect of herding on the whole ecological system and/or evolution groups. For physics, Section 4 not only presents the existence of phase transition in such a complex adaptive system, but also proposes a new equilibrium theory. Namely, in the presence of symmetry breaking, a complex adaptive system is likely to reach the equilibrium state only through cluster performance after the elements that construct the system form typically sized clusters.

5. Contrarian behavior

In Section 4, I have introduced the role of herd behavior. Here "herd behavior" means following majority. In Section 5, I want to raise and answer a connected question: what if one follows minority, instead of majority? Following minority actually corresponds to contrarian behavior, the inverse behavior of herd.

For engaging in competitions for sharing resources, agents in complex adaptive systems (CASs) often utilize various kinds of strategic behaviors, one of which is contrarian behavior. Contrarian behavior means figuring out what the herd is doing, and doing the opposite [84]. Contrarian behavior can be regarded as a kind of self-organization, which is one of the characteristics which distinguish CASs [85] from other types of complex systems. To dig out the nature of contrarian behavior is also of practical importance when one faces the relevant problems of resource allocation, say, risk evaluation and crisis management. Thus, contrarian behavior has been an active subject of studies in various fields like finance/economics [86], complexity science [87], and social science [88–91]. In social field, previous contrarian studies using Galam model of two state opinion dynamics [88–90] aimed at the effect of contrarian choices on the dynamics of opinion forming, which shed a significant light on hung elections. In Section 5, we designed to study the effect of contrarian behavior on social resource allocation. It is a common belief that contrarian behavior always stabilizes resource allocation by shrinking the redundancy or the lack of resources (positive role). However, is this common belief true? Here we specially raise this question because unbiased or biased distributions of resources are everywhere in Nature where contrarians are often needed. In other words, to comply with the real world, we need to investigate the role of contrarian behavior as the environment (which here is defined by the ratio between two resources, namely, resource ratio) varies.

The above-mentioned CASs involving competitions of agents for various kinds of resources can be modeled as a typical class of artificial well-regulated market-directed resource-allocation systems (simply denoted as "resource-allocation systems'' in the following) [21,64], as an extension of the original minority game [7]. Such resource-allocation systems can reflect some fundamental characteristics of the above CASs in the real world [27,7,8,21,64], say, a resource-allocation balance emerged as a result of system efficiency [21,64]. Thus, without loss of generality, we shall investigate the role of microscopic agents' contrarian behavior in the macroscopic properties of the resource-allocation system. In the process, we shall identify a class of transition points which help to distinguish the positive role (stabilizing, etc.) and the negative role (unstabilizing, etc.) of contrarian behavior for an unbiased/a weakly biased and a strongly biased distribution of resources, respectively. Comparing with the contrarian study by Galam [88] which also shows the transition point at a critical value of the contrarian proportion to identify opinion group forming, here, the transition points (in Section 5) help us to reveal that the allocation of resources can be optimized at the transition point by adding an appropriate size of contrarians which is observed in human experiments. To proceed, based on the extensively-adopted approaches of both statistical analysis [92,5,93,94] and agent-based modeling [7,8,21,64,1,95], we shall resort to three complementary tools: human experiments (producing data for statistical analysis), heterogeneous-agent-based computer simulations (of agent-based modeling), and statistical-mechanics-based theoretical analysis (of agent-based modeling).

5.1. Controlled experiments

We designed and conducted a series of computer-aided human experiments on the basis of the resource-allocation system [27,7,8,21,64]. As revealed in [21,64], the system can reach macroscopic dynamic balance that corresponds to the most stable state where the resources are allocated most efficiently and the total utilities of the system are maximal due to the absence of macroscopic arbitrage opportunities. Here we add a proportion of contrarians, in order to observe how contrarian behavior affects the macroscopic properties of the resource-allocation system. For the experiments, we recruited 171 subjects, all of which are students and teachers from several departments of Fudan University. The experiments were conducted in a big computer laboratory, and each subject had a computer to work with. All of the subjects were given a leaflet interpreting how the experiment would be performed before the experiment started. In the computer-aided online experiment, there are two virtual rooms: Room 1 and Room 2. Each room owns a certain amount of resources marked as M_1 or M_2 accordingly. The subjects do not know the exact resource ratio, M_1/M_2, at every experimental round. In the experiment, any kind of communication is not allowed, and every subject chooses to enter Room 1 or Room 2 independently to share the resources in it. Meanwhile, the computer program secretly adds contrarians into the system whose behaviors are controlled by the following settings. In every round of the experiment, each contrarian randomly chooses 5 subjects as his/her group. And then the contrarian will choose to enter the less-entered room according to the group. For example, if most of the subjects in a contrarian's group choose to enter Room 1, the contrarian will choose to enter Room 2. The total number of the subjects and the contrarians entering Room 1 and Room 2 are denoted as N_1 and N_2, respectively. After every experimental round, if $M_1/N_1 > M_2/N_2$, we say Room 1 (or Room 2) is the winning (or losing) room, because the subjects and contrarians entering Room 1 obtain more resources per capita, and vice versa. The subjects in the winning room will be granted 10 scores, and those in the losing room will be given 0 score. The final rewards are based on the scores each subject obtains in all the experimental rounds according to the exchange rate: 10 scores = 1 Chinese Renminbi. Besides, we will pay every subject 30 Chinese Renminbi as the attendance fee, and reward the top 10 subjects (having the highest scores), each with extra 100 Chinese Renminbi. More details are explained in the Appendix of Ref. [65].

In the experiment, we adjusted two parameters: one is the resource ratio, M_1/M_2, and the other is the ratio between the number of contrarians and subjects, β_c. 30 experimental rounds were repeated under each parameter set: M_1/M_2 and β_c. Let us denote the number of subjects as N_n and the number of contrarians as N_c, thus yielding $\beta_c = N_c/N_n$. In addition, the total number of all the subjects and contrarians is $N = N_n + N_c = N_1 + N_2$.

The experiment was conducted in two successive days: 88 subjects on the first day and 83 on the second day. The different number or different subjects show no influence on the results of the experiment. The experimental results are shown in Fig. 18 where $\langle N_1\rangle/\langle N_2\rangle$ is plotted as a function of M_1/M_2. When the distribution of resources is weakly biased up to $M_1/M_2 = 3$, the experimental results of $\langle N_1\rangle/\langle N_2\rangle$ are approximately located on the line with slope = 1 for the three values of β_c. In such cases, the system reaches dynamic balance at which the total utilities of the system are maximal due to the elimination of the macroscopic arbitrage opportunities. Nevertheless, for the strongly biased resource ratio, say $M_1/M_2 = 10$, the balance is broken as shown by the three experimental values that deviate far from the ''slope = 1'' line. In other words, as the resource ratio is unbiased or weakly biased, adding a small proportion of contrarians does not hurt the system balance. In contrast, as the resource ratio is biased enough, the contrarians of the same proportion break the balance instead.

Then, we analyze the experimental results from both individual and overall aspects of preference. As we know, different individuals have different preferences to a resource, which reflects heterogeneity of preferences. The heterogeneity has a remarkable influence on achieving the balance of the system. Here, the preference of each subject is defined as his/her average rate of entering Room 1 in the 30 rounds of experiments. The statistical results are shown in Fig. 19. Fig. 19(a) shows the result for $M_1/M_2 = 1$ and $\beta_c = 0$. The preferences of the subjects are different albeit of the unbiased distribution of the two resources, $M_1/M_2 = 1$. We see that the third subject preferred Room 1 while the second subject preferred Room 2. Such heterogeneity of preferences remains after introducing contrarians in Fig. 19(b–c). As for the larger resource ratios in Fig. 19(d–f) and Fig. 19(g–i), the subjects still have different preferences. However, the average preference of all the subjects varies with M_1/M_2, which illustrates the environmental adaptability of the subjects.

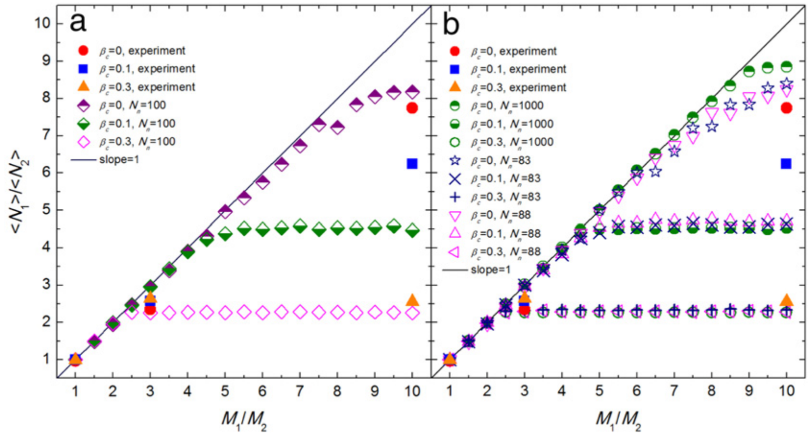

Fig. 18. Population ratio, $\langle N_1\rangle/\langle N_2\rangle$, as a function of resource ratio, M_1/M_2. The line with "slope $=$ 1" indicates the balance state where $\langle N_1\rangle/\langle N_2\rangle = M_1/M_2$. Each experiment lasts for 30 rounds (the first 6 rounds for equilibration and the last 24 rounds for statistics). Simulations are run for 400 time steps (the last 200 time steps for statistics and the first 200 time steps for equilibration). In (a), the number of normal agents in simulations is 100. In (b), the numbers of normal agents are respectively 1000, 83, and 88. $\langle\cdots\rangle$ denotes the average over the last 24 rounds for the experiment or the last 200 time steps for the simulations. The three experimental data at $M_1/M_2 = 1$ are overlapped. Parameters for the simulations: $S = 8$ and $P = 64$.
Source: Adapted from Ref. [65].

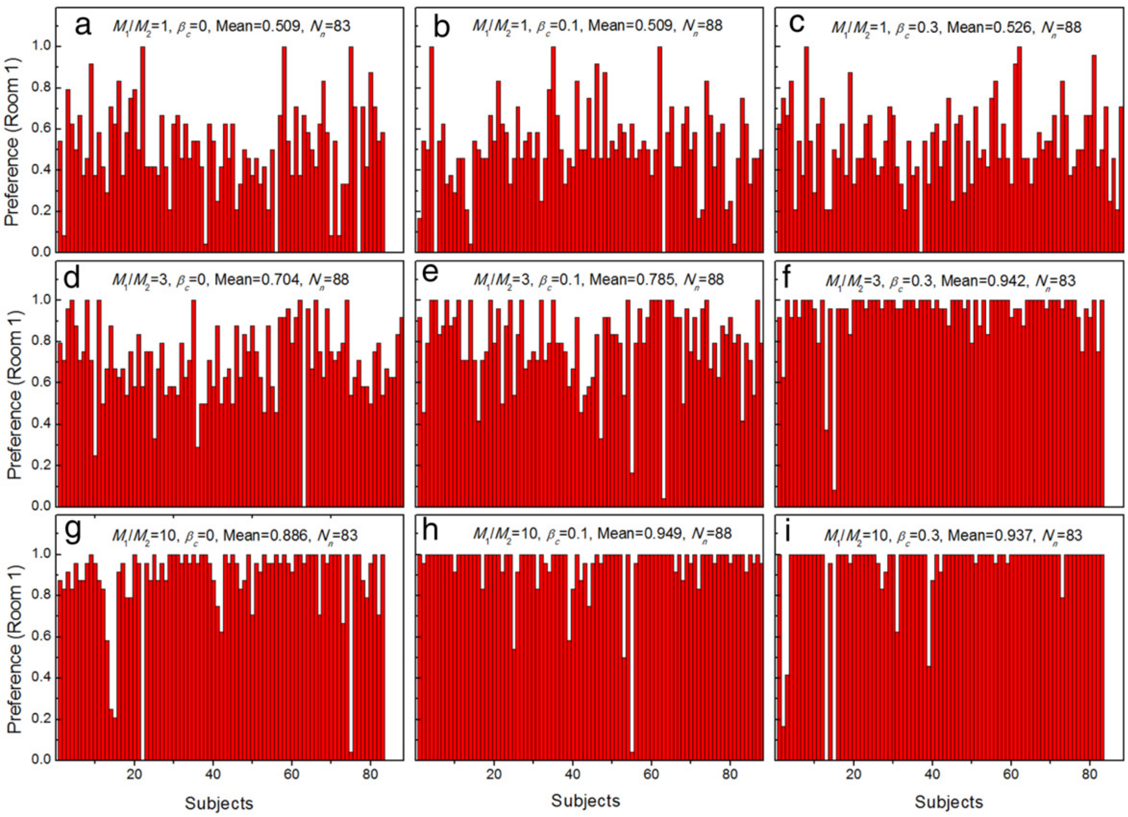

Fig. 19. Experimental data of the preference of each subject to Room 1 for the nine parameter sets with $M_1/M_2 = 1$ (a–c), 3 (d–e), and 10 (g–i) and $\beta_c = 0$ (a, d, g), 0.1 (b, e, h), and 0.3 (c, f, i). For each parameter set, the experiment lasts for 30 rounds (the first 6 rounds for equilibration and the last 24 rounds for statistics). In the figure, "Mean" denotes the average preference of all the subjects.
Source: Adapted from Ref. [65].

Next, in order to clearly observe the influence of contrarians on the macroscopic system, we calculated the stability of the system, $f = \frac{1}{2N}\sum_{i=1}^{2}\langle(N_i - \tilde{N}_i)^2\rangle$ [21], where $\langle\cdots\rangle$ denotes the average of time series $\cdots$. This definition describes the fluctuation in the room population away from the balance state at which the optimal room population, $\tilde{N}_i = M_iN/(M_1+M_2)$, can be realized. Clearly, the smaller value of f is, the closer the system approaches to the dynamic stability. Fig. 20(a)

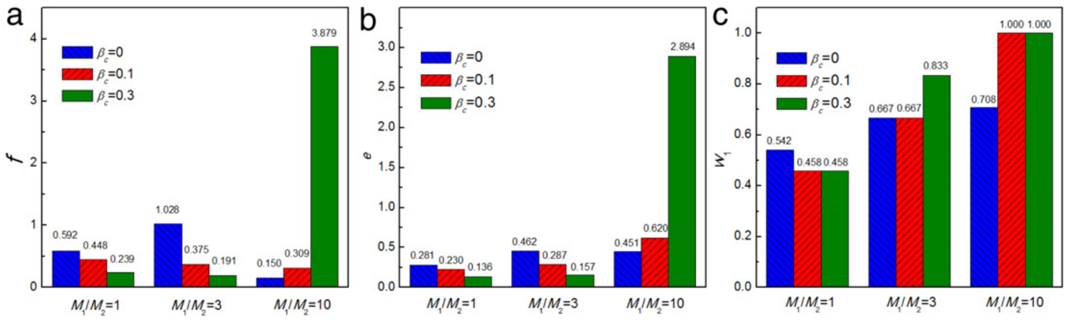

Fig. 20. Experimental data for (a) stability f, (b) efficiency e, and (c) predictability w_1 at $M_1/M_2 = 1$, 3, and 10. Each experiment lasts for 30 rounds (the first 6 rounds for equilibration and the last 24 rounds for statistics).
Source: Adapted from Ref. [65].

displays that, for small M_1/M_2, the fluctuations of the system decrease after introducing contrarians. Namely, the system becomes more stable. However, for large M_1/M_2, adding contrarians makes the system more unstable. Thus, we generally conclude that M_1/M_2 has a threshold, which distinguishes the different role of contrarians in the stability of the system. This experimental phenomenon will be further interpreted in the following part of computer simulations and theoretical analysis about transition points.

To further evaluate the performance of the overall system, we have also calculated the efficiency and the predictability of the resource-allocation system. Here, the efficiency is defined as $e = \left|\frac{\langle N_1\rangle}{\langle N_2\rangle} - M_1/M_2\right| / (M_1/M_2)$ [21]. Evidently, a larger value of e means a lower efficiency of resource allocation, and vice versa. Fig. 20(b) shows the change of e when adding contrarians into the experiment. When M_1/M_2 is 1 or 3, the adding of contrarians makes the resource-allocation system more efficient. However, for $M_1/M_2 = 10$, the presence of contrarians reduces the efficiency. Fig. 20(c) shows the predictability of the system which is represented by the winning rate of Room 1, w_1 [64]. Note $w_1 = 0.5$ means the winning rate is the same for both Room 1 and Room 2, which is hard for the subjects to predict. If w_1 deviates from 0.5, the winning rate of one room is higher than the other, so the subjects can predict the results easily. According to Fig. 20(c), when $M_1/M_2 = 1$, the winning rate w_1 fluctuates around 0.5 which means it is hard to do the prediction. But if M_1/M_2 becomes larger, the subjects are easy to predict the winning room for the next round especially when enough contrarians are added.

5.2. Agent-based modeling

Clearly the above experiment has some unavoidable limitations: specific time, specific experiment avenue (a computer room in Fudan University), specific subjects (students and teachers of Fudan University), and the limited number of subjects. Now we are obliged to extend the experimental results (Figs. 18–20) beyond such limitations. For this purpose, we establish an agent-based model on the basis of the resource-allocation system. In this model, we denote N_n as normal agents and N_c as contrarians. Normal agents correspond to the subjects in the experiment and each of them decides to enter one of the two rooms using their strategy table which is the same as the one designed in the agent-based model of market-directed resource-allocation game [21,64]. Particularly, the table of a strategy is constructed by two columns. The left column represents P potential situations and the right column is filled with 0 and 1 according to the integer, L, which characterizes the heterogeneity in the decision-making of normal agents. For a certain value of L, ($L \in [0, P]$), there is a probability of L/P to be 1 in the right column of the table and a probability of $(P - L)/P$ to be 0. Here 0 and 1 represent entering Room 2 and Room 1, respectively. At each time step, normal agents choose to enter a room according to the right column of the strategy tables directed by the given situation P_i, ($P_i \in [1, P]$). Before the simulation starts, every normal agent will randomly choose S strategy tables, each determined by an L. At the end of every time step, each normal agent will score the S strategy tables by adding 1 (or 0) score if the strategy table predicts correctly (or incorrectly). Then, the strategy table with the highest score will be used for the next time step. In addition, because contrarians have no strategy tables, their behavior is set to be the same as that already adopted in the experiment.

5.3. Simulation results

For the computer simulations, we use 100 normal agents and set $S = 8$ and $P = 64$. The result of $\langle N_1\rangle/\langle N_2\rangle$ versus M_1/M_2 is shown in Fig. 18(a). Clearly, qualitative agreement between experiments and simulations is displayed. In order to confirm this result, we conduct more simulations with different numbers of normal agents to compare with experimental results, which are shown in Fig. 18(b). We choose to use 83 and 88 normal agents which are consistent with experiments, and 1000 normal agents which represent the case of a remarkable different size. Comparing the different simulations in Fig. 18(a) and (b), their results show no qualitative differences though the number of normal agents varies. Therefore, we can say that the number of agents has no influence on our simulation results. This means that the experimental results reported in Fig. 18

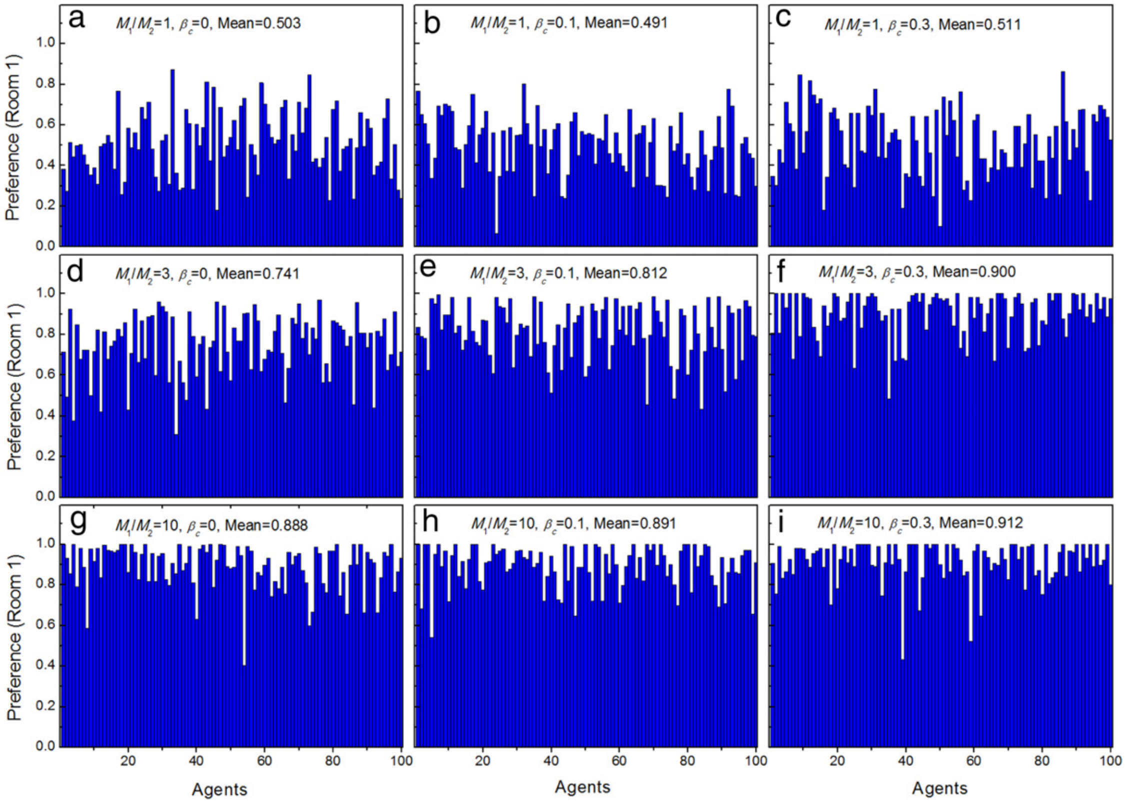

Fig. 21. Simulation data of the preference of each normal agent to Room 1 for the nine parameter sets with $M_1/M_2 = 1$ (a–c), 3 (d–e), and 10 (f–i) and $\beta_c = 0$ (a, d, g), 0.1 (b, e, h), and 0.3 (c, f, i). For each parameter set, simulations are run for 400 time steps (the last 200 time steps for statistics and the first 200 time steps for equilibration). In the figure, "Mean" denotes the average preference of the 100 normal agents.
Source: Adapted from Ref. [65].

are general (at least to some extent), being beyond the above-mentioned experimental limitations. Thus, we are confident to do more simulations in the following. For convenience, we use 100 normal agents in the remainder of Section 5.

In order to compare with the experiment, the preferences of 100 normal agents are also calculated; see Fig. 21. The simulation results are very similar to the experimental results in Fig. 19. That is, normal agents also show the heterogeneity of preferences and the environmental adaptability.

Then we are in a position to scrutinize the role of contrarians. To compare with the experimental results in Fig. 20, we also calculate stability f, efficiency (e) and predictability (w_1); see Fig. 22.

From Fig. 22, we find that the resource-allocation system clearly exhibits a transition point when taking M_1/M_2 and w_1 as the tuning parameter and order parameter, respectively. This echoes with what we have reported in [64]. In the mean time, at the transition point, $(M_1/M_2)_t$, f reaches the lowest value which means the system becomes the most stable. In details, for a small β_c, increasing M_1/M_2 will increase the system stability until f has the minimum value at $(M_1/M_2)_t$, which corresponds to the most stable state of the system. Once the minimum value is passed, the stability of the system will worsen for larger M_1/M_2. The former (or the latter) is positive (or negative) role of contrarians. As for large β_c, increasing M_1/M_2 will always make the system more unstable (negative role). Besides, as β_c increases, $(M_1/M_2)_t$ moves towards the direction of decreasing M_1/M_2. We shall discuss the movement of $(M_1/M_2)_t$ in the following theoretical analysis.

Fig. 22(b) shows the simulation results for the change of system efficiency, e. When M_1/M_2 is small, increasing contrarians can make the system more efficient at a certain range. In contrast, for large M_1/M_2, adding contrarians always reduces the efficiency. Such simulation results echo with those experimental results as shown in Fig. 20(b).

Fig. 22(c) displays the predictability of Room 1. Similarly, we can see from Fig. 22(c) that when M_1/M_2 is very small (close to 1), the winning rate of two rooms remains almost unchanged at 0.5 or so, even though β_c varies. That is, in this case, the system is unpredictable. When M_1/M_2 is gradually increasing, adding more contrarians will cause w_1 to increase from the value for $\beta_c = 0$; namely, it becomes more easy for the agents to predict the winning room. Again, these simulation results agree with those experimental results in Fig. 20(c).

Now, we can understand the role of contrarians in the resource-allocation system. On one hand, contrarians have positive roles as M_1/M_2 is small. Namely, adding contrarians can help to not only improve the system stability, but also increase the system efficiency while keeping the system unpredictable. On the other hand, contrarians have negative roles as M_1/M_2 becomes large enough. That is, adding contrarians can hurt the system stability and efficiency while making the system

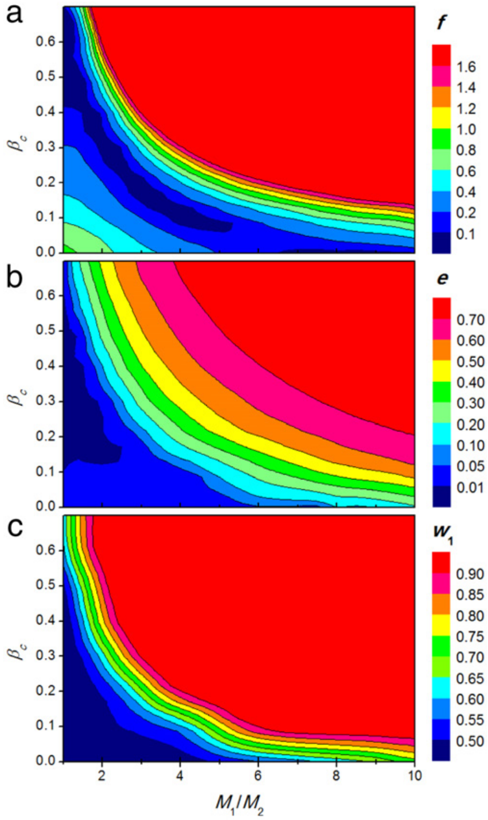

Fig. 22. $\beta_c - M_1/M_2$ contour plots for (a) stability f, (b) efficiency e, and (c) predictability w_1. For each parameter set, simulations are run for 400 time steps (the last 200 time steps for statistics and the first 200 time steps for equilibration).
Source: Adapted from Ref. [65].

more predictable. Both positive and negative roles have been well distinguished by identifying a transition point, $(M_1/M_2)_t$. Further, it is clear that the transition points identified herein also help to reveal that the allocation of resources can be optimal (i.e., stable, efficient, and unpredictable) at $(M_1/M_2)_t$ by adding an appropriate size of contrarians.

5.4. Theoretical analysis

In order to get a better understanding of the underlying mechanics of the agent-based model, we conduct theoretical analysis. When S and P are fixed, the system of our interest could reach the most stable state only at the transition point, i.e., a particular ratio between the two resources, $\left(\frac{M_1}{M_2}\right)_t$. If we adjust the values of β_c, the transition point, $\left(\frac{M_1}{M_2}\right)_t$, will change accordingly.

A. The properties of the transition point, $\left(\frac{M_1}{M_2}\right)_t$

(a) Without contrarians: It can be proven that for the agent-based model, the transition point has two properties: (1) every normal agent uses the strategy with the largest preference, $(L_i)_{\max}$, in his/her hand; (2) the system is in the balance state, which means the ratio between the numbers of agents in the two rooms is equal to the ratio between the two resources [21]. We first define

$$N_1 = \sum x_i,$$

where the choice of agent i is denoted as $x_i = 1$ (Room 1) or 0 (Room 2). Then, at the transition point, the expected ratio of normal agents who chooses to enter Room 1 is

$$\frac{\langle N_1 \rangle}{N_n} = \frac{\sum \langle x_i \rangle}{N_n} = \frac{\sum_i^{N_n} (L_i)_{\max}}{PN_n} = \left(\frac{M_1}{M_1 + M_2}\right)_t, \quad (4)$$

where $\langle \cdots \rangle$ denotes the averaged value of $\cdots$. Eq. (4) shows that when $\frac{M_1}{M_1+M_2} > \left(\frac{M_1}{M_1+M_2}\right)_t$, Room 1 will become unsaturated. This means the system does not stay at the balance state.

(b) With contrarians: From the properties of the transition point and the behavior of the contrarians, it can be shown that, all the normal agents still use the largest-preference strategy $(L_i)_{\max}$ at the transition point when contrarians are added. Every contrarian follows the minority in his/her group to make a choice denoted as x_c. Then the expected ratio of agents (both normal agents and contrarians) who choose to enter Room 1 at the transition point becomes

$$\frac{\langle N_1\rangle}{N} = \frac{\sum_i^{N_n}(L_i)_{\max} + P\sum_c^{N_c}\langle x_c\rangle}{(1+\beta_c)PN_n} = \left(\frac{M_1}{M_1+M_2}\right)_{t'}, \tag{5}$$

where $\beta_c = \frac{N_c}{N_n}$ and $\left(\frac{M_1}{M_1+M_2}\right)_{t'}$ stands for the new transition point with contrarians added.

B. Finding the expressions of $\sum_i^{N_n}(L_i)_{\max}$ and $\sum_c^{N_c}\langle x_c\rangle$

(a) Without contrarians: The probability that L_i takes a certain integer from the range 0 to P is $\frac{1}{P+1}$. Then, the probability of $(L_i)_{\max}$ being a certain value of L is

$$p(L) = \left(\frac{L+1}{P+1}\right)^S - \left(\frac{L}{P+1}\right)^S.$$

If N_n is large enough, there is

$$\sum_i^{N_n}(L_i)_{\max} = \sum_{L=0}^{P} N_n p(L) L = PN_n\left[1 - \frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S\right], \tag{6}$$

In the absence of contrarians, the substitution of Eq. (6) into Eq. (4) leads to

$$\frac{\langle N_1\rangle}{N_n} = 1 - \frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S = \left(\frac{M_1}{M_1+M_2}\right)_t \equiv m_n, \tag{7}$$

where m_n represents the transition point for the system with only normal agents.

(b) With contrarians: Since the normal agents still use their strategy with $(L_i)_{\max}$ at the transition point after adding contrarians into the resource-allocation system. Therefore, for normal agents, we have

$$\frac{\langle N_{n1}\rangle}{N_n} = 1 - \frac{1}{P}\sum_{L=1}^{P}\left(\frac{L}{P+1}\right)^S = \left(\frac{M_1}{M_1+M_2}\right)_t \equiv m_n.$$

When contrarian c chooses k normal agents as his/her group, the probability to get a normal agent who chooses Room 1 can be expressed approximately as $\frac{\langle N_{n1}\rangle}{N_n} = \left(\frac{M_1}{M_1+M_2}\right)_t \equiv m_n$. Then, the probability for $x_c = 1$ (or 0) is

$$\sum_{q=0}^{y} C_k^q\left[\left(\frac{M_1}{M_1+M_2}\right)_t\right]^q\left[\left(\frac{M_2}{M_1+M_2}\right)_t\right]^{k-q} \equiv m_c \quad (\text{or} \quad 1-m_c),$$

where $\left(\frac{M_2}{M_1+M_2}\right)_t = 1 - m_n$, $y = \frac{k-1}{2}$, and k is odd. Thus, we have the average of x_c, $\langle x_c\rangle$, as

$$\langle x_c\rangle = m_c = \sum_{q=0}^{y} C_k^q (m_n)^q (1-m_n)^{k-q}. \tag{8}$$

Plugging Eq. (8) into Eq. (5) yields

$$\frac{\langle N_1\rangle}{N} = \frac{PN_n m_n + PN_c m_c}{(1+\beta_c)PN_n} = \left(\frac{M_1}{M_1+M_2}\right)_{t'},$$

and then we have

$$\frac{\langle N_1\rangle}{N} = \frac{m_n + \beta_c m_c}{1+\beta_c} = \left(\frac{M_1}{M_1+M_2}\right)_{t'}. \tag{9}$$

Clearly, by adjusting β_c, we can change the transition point of the resource-allocation system. Fig. 23 shows the monotonically decreasing trend of $\left(\frac{M_1}{M_2}\right)_t$ for increasing β_c, which displays an excellent agreement between theoretical and simulation results.

In both experiments and computer simulations, we have found that when the system is in the balance state $[M_1/M_2 < (M_1/M_2)_t]$, the fluctuations of the system decrease after introducing a small number of contrarians. Because in both experiments and simulations, the behavior of contrarians is set to follow the same rule, it is necessary to further analyze

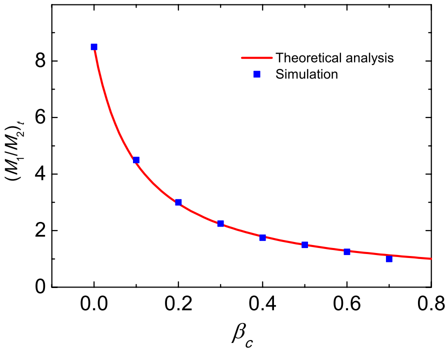

Fig. 23. Transition point $(M_1/M_2)_t$ versus β_c, as a result of theoretical analysis (curve obtained according to Eq. (9)) and simulation (data extracted from Fig. 22(a)). Parameters: $S = 8$ and $P = 64$.
Source: Adapted from Ref. [65].

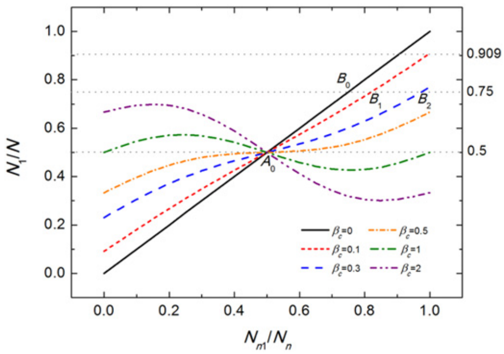

Fig. 24. N_1/N versus N_{n1}/N_n according to Eq. (10). The three horizontal gray dot lines are given by $N_1/N = 0.5$, 0.75, and 0.909, which are respectively related to the balance state of three resource ratios, $M_1/M_2 = 1$, 3, and 10.
Source: Adapted from Ref. [65].

the influence of this behavior on the stability of the whole system. Eq. (8) describes the probability of contrarians choosing to enter Room 1 when the system reaches balance. It is known that at this balance state, the number of subjects in the experiments (or normal agents in the simulations) choosing to enter each room still varies at every time step due to fluctuations. Hence we replace m_n in Eq. (8) with N_{n1}/N_n and get $\langle x_c\rangle = \sum_{q=0}^{y} C_k^q (N_{n1}/N_n)^q (1 - N_{n1}/N_n)^{k-q}$, where N_{n1} is the number of subjects or normal agents who choose to enter Room 1, and the average of x_c, $\langle x_c\rangle$, represents the expected probability of contrarians choosing Room 1. Note that $\langle x_c\rangle$ is a random variable due to the fluctuations of N_{n1}. Then, according to Eq. (9), we obtain

$$\frac{N_1}{N} = \frac{N_{n1}/N_n + \beta_c \langle x_c\rangle}{1+\beta_c}. \qquad (10)$$

By drawing N_1/N versus N_{n1}/N_n, we achieve Fig. 24, which shows the influence of the deviations of N_{n1}/N_n on N_1/N under different values of β_c. For $M_1/M_2 = 1$, it is shown that the balance point of the system lies on A_0 (0.5, 0.5) when $\beta_c = 0$, 0.1, 0.3, and 0.5. And the deviations of N_{n1}/N_n can cause the system to vibrate around A_0 along a certain line in Fig. 24 which is determined by β_c. Then, Fig. 24 shows that, under the same range of deviations of N_{n1}/N_n, by increasing β_c, we can bring down the vibration of N_1/N around $N_1/N = 0.5$. In addition, we can see from Fig. 24 that, when β_c becomes too large, such as $\beta_c = 1$ or 2, A_0 is no longer a stable point. The state of the system tends to move to the right end of the associated line because now more subjects or normal agents choosing to enter Room 1 will make Room 1 easier to win. That is, when β_c is too large, adding more contrarians will lead the system to a more unstable state. For a biased distribution of resources, say, $M_1/M_2 = 3$, Fig. 24 shows that the balance point of the system lies on different points for different values of β_c, i.e., B_0 (0.75, 0.75), B_1 (0.82, 0.75), and B_2 (0.97, 0.75) for $\beta_c = 0$, 0.1, and 0.3. It can be shown that adding a small number of contrarians makes the system with a biased distribution of resources more stable due to the following two reasons: (1) under the same deviations of N_{n1}/N_n, the vibration of N_1/N (say, around B_0, B_1, or B_2 for $M_1/M_2 = 3$) decreases slightly when adding more contrarians; (2) when adding more contrarians, the values of N_{n1}/N_n at the balance points (e.g., B_0, B_1, and B_2 for $M_1/M_2 = 3$) increase; in this case, subjects or normal agents will be more certain to choose Room 1, which reduces the deviation range of N_{n1}/N_n, thus decreasing the vibration of N_1/N.

5.5. Concluding remarks

In summary, using the three tools, we have investigated the role of contrarian behavior in a resource-allocation system. In contrast to the common belief that contrarian behavior always plays a positive role in resource allocation (say, stabilizes resource allocation by shrinking the redundancy or the lack of resources), the transition points have helped us to reveal that the role of contrarian behavior in resource-allocation systems can be either positive (to stabilize the system, to improve the system efficiency, and to make the system unpredictable) or negative (to unstabilize the system, to reduce the system efficiency, and to make the system predictable) under different conditions. Further, the transition points identified herein have also helped us to show that resource allocation can be optimized by including an appropriate size of contrarians.

Section 5 is also expected to be of value to some other fields. In management and social science, administrators should not only conduct contrarianism when finding the formation of herd, but also need to consider system environment and timing to see whether contrarianism is globally positive or negative. In ecology and evolution, it is not only necessary to study the mechanism of contrarian formation, but also to pay more attention to the effect of contrarianism on the whole ecological system and evolution groups.

6. Spontaneous cooperation

To study the collective phenomena in markets, I have introduced herd behavior and contrarian behavior in Sections 4 and 5, revealing some interesting results. In Ref. [96], the authors investigated a similar human system with the co-existence of both herd behavior and contrarian behavior (called hedge behavior [96,97]), and found that the systems with different numbers of investors could be statistically equivalent to each other. But, what if the same human system lacks either herd behavior or contrarian behavior? The answer can be found in Section 6, where the invisible hand, coined by A. Smith (June 5, 1723–July 17, 1790) [24], can come to appear spontaneously.

As mentioned in Sections 4 and 5, most of the social, economic and biological systems involving a large number of interacting agents can be regarded as complex adaptive systems (CAS) [85], because they are characterized by a high degree of adaptive capacities to the changing environment. The interesting dynamics and phase behaviors of these systems have attracted much interest of physical scientists. A number of microscopic CAS models have been proposed [98–100,80,101], among which the minority game (MG) [27,7,81] becomes a representative model. Along with the progress in the research of econophysics [5], MG has been mostly applied to simulate one kind of CAS, namely the stock market [29,28]. Alternatively, MG can also be interpreted as a multi-agent system competing for a limited resource [31,30] which distributes equally in two rooms. However, agents in the real world often have to face a competition to the limited resource, which distributes in different places in a biased manner. Examples for such phenomena include companies competing among markets of different sizes [102], drivers selecting different traffic routes [103], people betting on horse racing with the odds of winning a prize and making decisions on which night to go to which bar [104].

From a global point of view, the ideal evolution of a resource allocating system would be the following: Although each agent would compete against others only with a self-serving purpose, the system as a whole could eventually reach a harmonic balanced state where the allocation of resource is efficient, stable and arbitrage-free (which means that no one can benefit from the "misdistribution" of the resource). Note that during the process of evolution to this state, agents could neither have been told about the actual amount of the resources in a specific place, nor could they have any direct and full communications, just as if there were an "invisible hand" [24] directing them to cooperate with each other. Then does this invisible hand always work? In practice, there is plenty of evidence that the invisible hand do have very strong directing power in places such like financial markets, though sometimes it does fail to work. Such temporary ineffectiveness implies that there must be some basic conditions required for the invisible hand to exert its full power. Through an experimental study and a numerical study with a market-directed resource allocation game (MDRAG, which is an extended version of the MG model), we found that agents equipped with heterogeneous preferences, as well as a decision-making capacity which matches with the environmental complexity, are sufficient for the spontaneous realization of such a harmonic balanced state.

6.1. Controlled experiments

To illustrate the system behavior, we designed and conducted a series of economic experiments, collaborated with the university students. In the experiments, 89 students from different (mainly physics, mathematics, and economics) departments of Fudan University were recruited and randomly divided into 7 groups (Group A–G, see Tables 2–5). The number of students in each group was just set for convenience and denoted by N in Tables 2–5. In the games played in the experiments, students were told that they have to make a choice among a number of rooms, in each round of a session, for sharing the different amounts of virtual money in different rooms. Students who get more than the global average, namely those belonging to the relative minority, would win the payoff. At the beginning of a session, subjects were told the number of rooms (2 or 3), and in some cases the different but fixed amount of virtual money in each room. In the following, M_i is used to denote the amount of virtual money in room i. A piece of global information, about the payoff in the preceding round in all rooms, is announced before a new round starts. In each round, the students must make their own choices without any kinds of communication. The payoff per round for a student in room i is 2 points if $M_i/N_i > \sum M_i/N$, and -1 point otherwise. Here

Table 2
Results of GAME-I.
Source: Adapted from Ref. [21].

Session	Group	Round	M_1	M_2	M_3	$\langle N_1 \rangle$	$\langle N_2 \rangle$	$\langle N_3 \rangle$
1	A ($N = 12$)	1–10	3	2	1	5.3	4.6	2.1
2	A ($N = 12$)	1–10	3	2	1	5.5	3.8	2.7
3	B ($N = 12$)	1–10	3	2	1	5.5	4	2.5
4	C ($N = 24$)	1–20	3	2	1	12.2	7.4	4.4
5	D ($N = 10$)	1–10	5	3	–	6.1	3.9	–
6	D ($N = 10$)	1–10	3	1	–	7.4	2.6	–

Table 3
Results of GAME-II.
Source: Adapted from Ref. [21].

Session	Group	Round	M_1	M_2	$\langle N_1 \rangle$	$\langle N_2 \rangle$
1	D ($N = 10$)	1–10	2	1	6.2	3.8
2	E ($N = 10$)	1–10	1	3	3.3	6.7
3	F ($N = 11$)	1–10	3	1	7.2	3.8
3	F ($N = 11$)	11–20	3	1	8.3	2.7
4	C ($N = 24$)	1–15	7	1	17.8	6.2
4	C ($N = 24$)	16–30	7	1	21.1	2.9

Table 4
Track of 11 subjects in Group F ($N = 11$) converging to $M_1/M_2 = 3$.
Source: Adapted from Ref. [21].

Round	N_1	N_2
1	5	6
2	9	2
3	4	7
4	6	5
5	6	5
6	7	4
7	7	4
8	8	3
9	10	1
10	10	1
11	8	3
12	10	1
13	9	2
14	7	4
15	9	2
16	7	4
17	7	4
18	9	2
19	8	3
20	9	2

Table 5
Results of GAME-III.
Source: Adapted from Ref. [21].

Session	Group	Round	M_1	M_2	$\langle N_1 \rangle$	$\langle N_2 \rangle$
1	G ($N = 10$)	1–5	3	1	5.4	4.6
1	G ($N = 10$)	6–10	3	1	8.2	1.8
1	G ($N = 10$)	11–15	3	1	7.	3.
1	G ($N = 10$)	16–20	3	1	7.	3.
1	G ($N = 10$)	21–25	1	3	7.8	2.2
1	G ($N = 10$)	26–30	1	3	4.2	5.8
1	G ($N = 10$)	31–35	1	3	2.8	7.2
1	G ($N = 10$)	36–40	1	3	2.6	7.4
1	G ($N = 10$)	41–45	1	3	2.4	7.6

N_i is the number of the students choosing room i. The total payoff of a student is the sum of payoffs of all rounds which will be converted to money payoffs in RMB with a fixed exchange rate. Since the organizational and statistic procedures were done by humans, one session of 10 rounds took roughly 20 min. More details can be found in the leaflet to the experiment with nine items:

1. A group of subjects are taking part in this experiment. The game situation is the same for each subject. In the experiments, any kinds of communication are not allowed.
2. At the beginning of the game, all of you will be told the kind of game (GAME-I, GAME-II or GAME-III), as well as the total number of the subjects (N), rooms (2 or 3), and play rounds, respectively.
3. In each round of the game, you have to choose and enter one of the rooms. The amount of virtual money in each room is different but fixed, represented by M_i ($i = 1, 2, \ldots$).
4. You will be told each M_i (only in GAME-I).
5. In each round, you can choose a room to share the virtual money in it and get your quota, M_i/N_i, if you select room i. Here N_i denotes the total number of subjects in room i.
6. You may make a new room choice in every round.
7. Your payoff per round: After the statistics of each round is done, you will receive a payoff which depends on the relation between your quota and the global average:

$$\text{payoff per round} = \begin{cases} 2 \text{ points}, & \text{if } M_i/N_i > \sum M_i/N \\ -1 \text{ point}, & \text{otherwise} \end{cases} .$$

8. Your information per round:
   - The current round number;
   - N_i of each room in the preceding round (only in GAME-I, announced by the game organizer);
   - Payoff (2 or -1) of each room in the preceding round (announced by the game organizer);
   - Your rooms chosen and payoffs got in the preceding game rounds (recorded by yourself);
   - Your cumulated payoffs (calculated by yourself).
9. The initial capital of each subject is 0 point. The exchange rate is 1 RMB per (positive) point.

Three kinds of games, GAME-I, GAME-II and GAME-III, have been investigated. GAME-II differed from GAME-I in the global information being announced. In GAME-I, both the resource distribution M_i and the current population N_i in room i were announced, while only payoffs (2 or -1) in each room of the current round were conveyed to subjects in GAME-II. Note that the environmental complexity was increased in GAME-II, since in order to win the game, subjects would have to predict other subjects' decisions, in the meantime, infer the actual amount of virtual money in different rooms. In GAME-III, the global information is the same as that of GAME-II, except an abrupt change of amount of virtual money is introduced during the play of the game without an announcement. (On the contrary, all the subjects have already been told that each M_i is unchanged.) No further information was given to the subjects.

Results of six sessions of GAME-I, four of GAME-II, and one of GAME-III are given in Tables 2–5. In Table 2, the results of GAME-I are listed, where the time average of the subject number in room i is represented as $\langle N_i \rangle$. As the data shown, a kind of cooperation seems to emerge in the game within 10 rounds. In particular, ratios of $\langle N_i \rangle$ converge to the ratios of M_i, implying that the system becomes efficient in delivering the resource even if it was distributed in a biased way. To the subjects, no room is better or worse in the long run, there is also no evidence that any of them could systematically beat the resource allocation "market". One might naively think that the system could evolve to this state only because the subjects knew the resource distribution prior to the play of the games, and the population in each room during the play. However, results of GAME-II show that this explanation could not be correct. As shown in Table 3, although subjects who know neither the resource distribution nor the current populations in different rooms, seem not to be able to adapt to the unknown environment during the first 10 or 15 rounds, eventually the relation $\langle N_1 \rangle / \langle N_2 \rangle \approx M_1/M_2$ is achieved again in groups C and F. For instance, Table 4 shows the track through which group F gradually found the balanced state under the environmental complexity $M_1/M_2 = 3$. Furthermore, the results of GAME-III support the conclusion of GAME-II, in which the system can reach this state even with an abrupt change of the unknown resource distribution during the play of the game, see the results of 21 to 45 rounds played by the G group in Table 5. It is surprising that subjects can "cooperate" even without direct communications as well as the information of the resource distribution. We can define the source of a force which drives the subjects to get their quota evenly as the "invisible hand" of the resource allocation market. In the sequel, however, we shall show that the effectiveness of this invisible hand relates to the heterogeneous preference and the adequate decision-making capacity of the subjects of the game.

6.2. Agent-based modeling

To find out the mechanism behind this adaptive system of resource distribution, two multi-agent models are used and their results are compared with each other. The first model is the traditional MG, while the second one is an extended MG called as Market-Directed Resource Allocation Game (MDRAG). MG and MDRAG have a common framework: There are N agents who repeatedly join a resource allocation market. The amounts of resource in two rooms are M_1 and M_2, respectively. Before the game starts, each agent will choose S strategies to help him/her make decision in each round of play. The strategy used in MG and MDRAG is typically a choice table which consists of two columns, as shown in Table 6. The left column is for the P possible economic situations, and the right side is filled with the corresponding room number, namely room 0 or room 1. Thus, if the current situation is known, an agent should immediately choose to enter the corresponding room. With a given P, there are totally 2^P different strategies. At each time step, based on a randomly given exogenous economic

Table 6
A typical strategy table.
Source: Adapted from Ref. [21].

Economic situations	Choices
1	0
2	1
3	1
...	...
P-1	0
P	1

state [105], each agent chooses between the two rooms with the help of the prediction of his/her best scored strategy. After everyone has made a decision, agents in the same room will share the resource in it. Agents who earn more than the global average $(M_1 + M_2)/N$ become the winners, and the room which they entered is denoted as the winning room. To a strategy in the game, a unit of score would be added if it had given a prediction of the winning room, no matter it was actually used or not.

On the other hand, MDRAG differs from MG in the strategy-building procedures. In traditional MG, agents "randomly" choose S strategies from the strategy pool of 2^P size. Here "randomly" means that each element of the right column of a strategy table is filled in with 0 or 1 equiprobably. By using this method, strategies of different preferences will have a binomial distribution. Here the preference of a strategy is defined as the tendency or probability with which a specific room will be chosen when the strategy is activated. For a large P, the number of 0 and the number of 1 in the right column are nearly equal. Hence globally there would be no preference difference among agents who uniformly pick up these strategies. In MDRAG, however, we use a new method to fill the strategy table in order to introduce heterogeneous preferences to the agents. First, K $(0 \le K \le P)$ denoting the number of 0 in the right column is randomly selected from the $P + 1$ integers. In other words, strategies with different preferences (different values of K) are chosen equiprobably from the strategy pool. Second, each element of the strategy's right column should be filled in by 0 with the probability K/P, and by 1 with the probability $(P-K)/P$. It is clear that a strategy with an all-zero right column can be picked up with the probability $1/(P+1)$ in MDRAG, while this could happen only with a probability of $1/2^P$ in the traditional MG and practically could never be chosen by any MG agents if $NS \ll 2^P$.

To make descriptions easier to understand, explanations of the model parameters are provided. The ratio M_1/M_2 represents the environmental complexity of the games. Note that if $M_1/M_2 = 1$, agents need only to worry about other people's decision. Assuming that room 1 always contains more resource, the trivial case will be $M_1/M_2 > N - 1$, since all the agents can easily find out that going to room 1 would be the right choice under this situation. On the other hand, when the ratio is set $1 < M_1/M_2 \le N - 1$, the larger this ratio is, the more difficult it would be for the market to direct the system to the ideal state. Other parameters concern with the decision-making capacity, which can be generalized into three elements (http://plato.stanford.edu/entries/decision-capacity/), namely, (i) the possession of a set of values and goals necessary for evaluating different options; (ii) the ability to communicate and understand information; (iii) the ability to reason and to deliberate about one's choices. The first element has already been built in both the models as the evaluation of the strategies with the minority-favorable payoff function. The second element relates to the model parameter P. Since the total number of possible situations is dependent on the completeness of the perception of the world, we relate it to the cognition ability. Finally, more strategies could be helpful if one needs to deliberate his/her choices of decisions, hence the strategy number S is related to the third element of the decision-making capacity, the ability of choice deliberation.

6.3. Results

Results of the economic experiments are compared with the simulation results of the traditional MG and MDRAG in Fig. 25. For each parameter set, we performed the simulation for 200 times. In each of these simulations, the code was run over 400 time steps. First half of the time evolution is for the equilibration of the system, while the remaining half is for doing the statistics. With a certain set of parameters ($S = 8$ and $P = 16$), MDRAG's results perfectly agree with the experimental data under a higher environmental complexity. In other words, agents in both experiments and MDRAG can be directed by the market to cooperate with each other, so that an efficient allocation of the biasedly distributed resource can be realized even without giving the agents full information or instructions. On the other hand, the traditional MG fails to reproduce the experimental results unless the distribution of resource is biased very weakly up to $M_1/M_2 = 3$. Note that MDRAG differs from MG solely in the introduction of heterogeneous preferences in the strategies; hence one may infer that the heterogeneity of agents' preferences is a significant factor to have the "invisible hands" to be effective. This argument is further supported by numerical experiments in the 3-room cases (with parameters $P = 24$, $N = 120$ and $S = 10$). Again, here MDRAG is superior to MG in bringing out the directing power of the market. Shown in Table 7, the ratio of $\langle N_1\rangle : \langle N_2\rangle : \langle N_3\rangle$ converges to $M_1 : M_2 : M_3$ only in the equilibrium states of MDRAG.

Fig. 25 also shows that the decision-making capacity, in particular the deliberation of choices (the parameter S), would be another factor having influence on the effectiveness of the invisible hand. Typically, as the environmental complexity (M_1/M_2) increases, both MG and MDRAG will deviate from the experimental results. Nevertheless, the problem of MG is

Table 7
Performances of MDRAG and MG in 3-room cases.
Source: Adapted from Ref. [21].

Resource			MDRAG			MG		
M_1	M_2	M_3	$\langle N_1\rangle$	$\langle N_2\rangle$	$\langle N_3\rangle$	$\langle N_1\rangle$	$\langle N_2\rangle$	$\langle N_3\rangle$
1	1	1	39.9	40.3	39.8	39.2	41.5	39.3
1	2	3	19.7	40.1	60.2	25.1	40.7	54.2
1	4	7	9.6	40	70.4	24.8	40.4	54.8
1	2	9	9.8	19.6	90.6	29.3	33.8	56.9

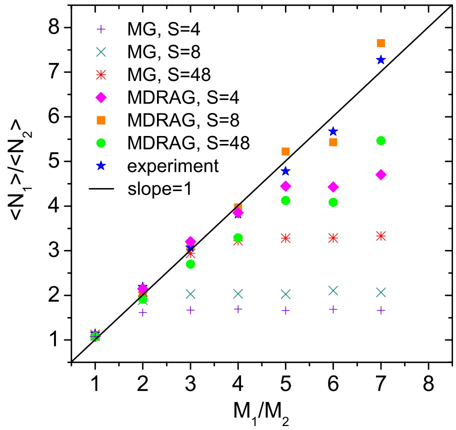

Fig. 25. $\langle N_1\rangle/\langle N_2\rangle$ as functions of M_1/M_2, $P = 16$ in MG and MDRAG, and $N = 24$ for all the simulations and the experiment. Simulations are run for 200 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). The line with slope = 1 indicates the efficient states: $\langle N_1\rangle/\langle N_2\rangle = M_1/M_2$.
Source: Adapted from Ref. [21].

much severe. As shown in the figure, even MG with extremely large S ($S = 48$, a situation which is inconsistent with the real system and will drastically increase the computational cost) can just work at a very low level of the environmental complexity. At the same time, the result of MDRAG provides a perfect fit with the experimental data when S is large enough, but not too large for a given P value (the reason will be explained in the following discussion). In one word, MG does not provide a good fit even for large S, while MDRAG can fit the data with less demanding condition in terms of computational cost.

6.4. Concluding remarks

Through a large number of numerical simulations, we have found the dependence of equilibrium states of the system on the model parameters, together with a number of phase transitions in the models. To explore in more detail, three parameters are defined which describe system behaviors in three aspects, namely, efficiency, stability and predictability. First the efficiency of resource allocation can be described as $e = |\langle N_1\rangle/\langle N_2\rangle - M_1/M_2|/(M_1/M_2)$. Note that $0 \le e < 1$ and a smaller e means a higher efficiency in the allocation of the resource. The stability of a system can be described by $\sigma^2/N \equiv \frac{1}{2N}\sum_{i=1}^{2}\langle(N_i - \tilde{N}_i)^2\rangle$, which denotes the population fluctuation away from the optimal state. [3] Here $\tilde{N}_i = M_iN/\sum M_i$, and $\langle A\rangle$ is the average of time series A_t. The predictability is related to $H \equiv \frac{1}{2NP}\sum_{\mu=1}^{P}\sum_{i=1}^{2}\langle N_i - \tilde{N}_i|\mu\rangle^2$, in which $\langle A|\mu\rangle$ is the conditional average of A_t given that $\mu_t = \mu$, one of the P possible economic situations. If $\sigma^2/N \neq H$, it means that agents may take different actions at different time for the same economic situation (namely the market behavior is unpredictable). For clarity, we describe the predictability of system by defining $J = 1 - HN/\sigma^2$. It is obvious that $0 \le J < 1$ and a smaller J means a higher predictability.

The variation of system behavior along with the change of environmental complexity M_1/M_2 is shown in Fig. 26. As shown in Fig. 26(a), the system changes from an efficient state into an inefficient state at some critical value $(M_1/M_2)_c \sim S$. For other values of P, the system behavior keeps the same as long as P is large than M_1/M_2. In Fig. 26(b), around the same critical value of M_1/M_2, σ^2/N changes from a decreasing function to an increasing function, giving the smallest fluctuation in the population distribution at the critical point. Meanwhile the order parameter J also falls into zero at $(M_1/M_2)_c$, suggesting that a phase transition, named the "M_1/M_2 phase transition", occurs at this critical point. To be more illustrative, when the environmental complexity is much smaller than the critical value, the system could reside in an efficient, unpredictable

3 Large fluctuations in populations can cause a higher dissipation in the system. Hence an efficient and stable state means an optimal state with a low waste in the resource allocation.

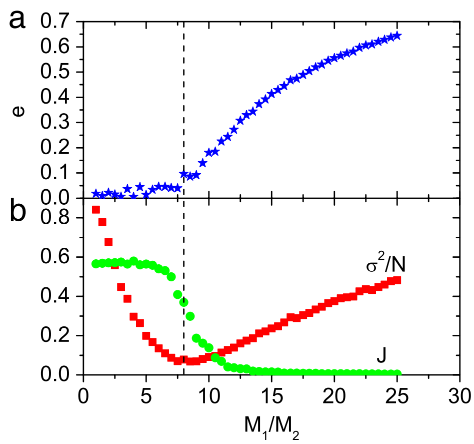

Fig. 26. The "M_1/M_2 phase transition" in MDRAG, for $N = 100$, $P = 64$, and $S = 8$. Simulations are run for 300 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). The dashed line denotes $M_1/M_2 = 8$. (a) e as a function of M_1/M_2. (b) σ^2/N and J versus M_1/M_2, respectively.
Source: Adapted from Ref. [21].

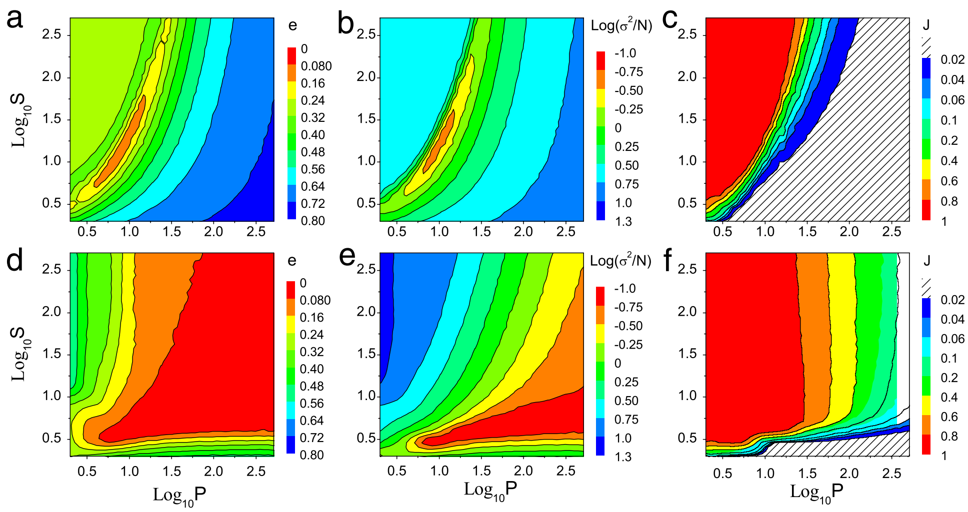

Fig. 27. S–P contour maps for different parameters on log–log scales with $N = 80$ and $M_1/M_2 = 4$. For each data point, simulations are run for 50 times, each over 400 time steps (first half for equilibration, the remaining half for statistics). (a), (b), and (c) are e, σ^2/N, and J as functions of S and P in MG, respectively. (d), (e), and (f) are e, σ^2/N, and J as functions of S and P in MDRAG, respectively. Regions filled with slash shadow denote the predictable states.
Source: Adapted from Ref. [21].

but relatively unstable state. Getting closer to the phase transition point, the stability of system will be improved until the most stable state is reached. Then after crossing the critical point, the decision-making capability of the whole system has been exhausted and it will fall into an inefficient, predictable, and unstable state. At the vicinity of the critical point, as if participants of the game worried about being eliminated from the competition, the market inspires all of its guiding potential and leads the system to the ideal state for the resource allocation, a state which is both efficient and stable and where no unfair arbitrage chance can exist.

It is important to know that MDRAG and MG have totally different phase structures, which could be analyzed by comparing the $S - P$ contours of the descriptive parameters for the two models, see Fig. 27. From the analysis, we could also know how the decision-making capacity influence the overall performance of the resource allocation system, in case that the environmental complexity is fixed ($M_1/M_2 = 4$). Features of the contour maps (Fig. 27) are summarized as the following (different M_1/M_2's do not change the conclusions):

(i) Comparing with the traditional MG as a whole, MDRAG has a much wider range of parameters for the availability of the efficient, stable and unpredictable states. In particular, there is almost no eligible region in Fig. 27(a) if we take the criterion of efficiency as $e < 0.08$. Also, the predictable region ($J < 0.02$) in Fig. 27(f) is much smaller than that of the MG's results in Fig. 27(c). These facts indicate that MDRAG has a much better performance than MG as a resource allocating system.

(ii) Patterns of the contour maps suggest that MG and MDRAG have totally different dependency on parameters. Fig. 27(a)–(c) indicate that P and S are not independent in the traditional MG model, which confirms the previous findings [8]. On the other hand, in MDRAG there is always a region where the system behavior is almost controlled by the parameter S. In Fig. 27(d), for large enough P, the system can reach the efficient state if S exceeds a critical value S_p, where S_p will converge to the limit value M_1/M_2 with increasing P. For $S < S_p$, the system can never reach the efficient state no matter how P changes. For a very large P and $S < M_1/M_2$, it can be proved that the probability for agents to enter the richer room is $S/(S+1)$, so that the system stays in the inefficient states ($\langle N_1\rangle/\langle N_2\rangle < M_1/M_2$).

(iii) Observing Fig. 27(d) and Fig. 27(f), one may find both an "S phase transition" and a "P phase transition". As mentioned above, for large enough P, the increase of S can abruptly bring the system from the inefficient/predictable phase to the efficient/unpredictable phase, and it is named "S phase transition". On the other hand, in the narrow region where $S < M_1/M_2$, the increase of P can also produce of a drastic change from the unpredictable phase to the predictable phase, and it is named "P phase transition". The existence of the "S phase transition" can be explained by the fact that the number of available choices in decision-making is a key factor for agents to find the right choice from strategies with an adequate heterogeneity of preferences. But for $S \gg P$, it will also cause a slight decrease of the efficiency because of the conflicts of the different predictions from the equally good strategies with the same preference. This explains why "MDRAG, $S = 48$" performs worse than "MDRAG, $S = 8$" when $P = 16$ in Fig. 25. The "P phase transition" reflects that for some incompetent ability of choice deliberation, a critical value of the cognition ability can enhance the decision-making capacity to match the environmental complexity.

(iv) It is also noteworthy that the parameter $\alpha = 2^p/N$, which is the main control parameter in the MG model [7,81], no longer controls on the behavior of the MDRAG system. Varying N while keeping M_1/M_2 as a constant, the basic feature, especially the critical position of the contour maps, will remain unchanged.

In aspects of the competition for resources, the feed of global information, and the inductive optimization of strategies, both MG and MDRAG may be regarded as eligible models for the economic experiments. However, MG fails to reproduce the experimental results in most cases. By simply accommodating a broader preference distribution of the strategies, MDRAG fits the experimental results without any coordinating capability of the agents. This enables us to comment on the possible mechanism of the "invisible hand", and conditions under which the complex adaptive systems will spontaneously converge to the efficient states. The most important thing for the "invisible hand" to work is that different subjects of the economic games should have different preferences, just like the agents in the MDRAG model who have heterogeneous preferences in their strategies. Next the subjects should also own an adequate capacity of decision-making which matches the complexity of the environment. From the M_1/M_2 phase transition in the MDRAG simulations, we could infer that there would be a failure in achieving the balanced efficient state if the game of experiment were designed too complicated, e.g. too many rooms or too biased distribution of the virtual money. Nevertheless, for the MDRAG model itself, since the model parameters can be tuned freely, we believe that the market directing power can always be brought out completely in this paradigm as long as the computational power is enough. To put it further, when the experiment happens to be set at the critical range of subjects' decision-making capacity, just like a finely tuned MDRAG where parameters are set to be critical values of the phase transitions mentioned above, an idealized state of the resource allocating system can be realized, namely, the system is efficient, stable and unpredictable. For example, see the overlapped regions for small e, small σ^2/N and finite J in Fig. 27(d)–(f).

Finally, although these intriguing conclusions are supported by the results of MDRAG simulations, there are still some important effects in the real world not included in the model, such as the difference in the decision-making capacities among the agents and the agents' responses to changes of the environment. One challenging task is to consider a suitable relation between agents' behavior and the distribution of resources (M_1/M_2) which may have an influence on the dynamic behavior of the whole system.

7. Partial information

In Sections 4–6, each subject always knows the overall competition result of all the subjects, which is defined as complete information herein. But, in reality, one often makes decisions according to the result of a part of a crowd (which is defined as partial information in Section 7), rather than the complete information. So, it becomes interesting to know what is the different effect of partial information. This is just the topic of Section 7.

It is well known in statistical physics that there exist a lot of phase transition phenomena, e.g., the melting of ice (classified as first-order phase transition) and the superfluid transition (classified as second-order phase transition; both second-order and higher-order phase transitions are also called continuous phase transitions) [35]. In complex adaptive systems, phase transition phenomena can be seen as well [21,81,106–108].

In economics and social science, it is a common belief that if more information is distributed to agents, the associated system will be more efficient (here "more efficient" means that the system is easier to lie in the optimal state and also the fluctuation level of the system is lower). For example, the famous Efficient Market Hypothesis [109] implies that market efficiency will be higher if more information is available to investors, i.e., the efficiency upgrading from the weak form to the strong form [109,110]. In Section 7, we recheck this information effect in a complex adaptive system related to resource-allocation problems [21,64,65,111,112,7] because these problems are of particular importance. For instance, some believe governments should get involved to guarantee that resources are distributed to places where they are needed most, while others think markets can use the invisible hand [24] to ensure the efficiency of resource allocation [113,24]. Both agent-based simulations and controlled human experiments have been adopted to discuss the problems. And in the associated artificial system, the invisible hand phenomenon emerges as well [21]. In the former research [21], all the subjects could get access to global information to evaluate their strategies. However, in many real problems, global information is either difficult to collect or confidential. For example, when a company decides whether or not to step into an emerging market, it is hard to know all the other competitive companies' reactions and also it would take a long time to learn about the real return on investment. Therefore, the company can only draw up its own strategies under currently obtained partial information. This leads to a question: when market participants obtain only partial information, can the invisible hand still work in the market?

To model this question, agents in our system are connected via a directed random network and everyone evaluates his/her own performance through partial information gathered from his/her first-order neighborhood. It is obvious that a higher connection rate can make the system more information-concentrated. Our agent-based model is designed on the basis of the market-directed resource allocation game [21], which can be used to simulate the biased or unbiased resource distribution problems. And we also conduct a series of controlled human experiments to show the reliability of the model design. We find that the system can reach the optimal (balanced) state even under a small information concentration. Furthermore, when the information concentration increases, the ensemble average of the fluctuation level goes through a continuous phase transition, which means that in the second phase, agents getting too much information will harm the system's stability (a higher fluctuation level means a lower stability of the system) instead. This is contrary to the belief mentioned above (namely, it will be better for the market efficiency when more information is shared). And at the transition point, the ensemble fluctuations of the fluctuation level remain at a low value. We also show that when the system becomes infinitely large, this fluctuation transition phenomenon remains. Our finding is in contrast to the textbook knowledge about continuous phase transitions, which states fluctuations will rise abnormally around a transition point since the correlation length becomes infinite. Thus, we call this continuous phase transition *anomalous continuous phase transition*.

7.1. Agent-based modeling

To proceed, let us first introduce the abstract resource allocation system of our interest. The system contains a repeated game. In the game, there are N agents facing two rooms labeled as Room 1 and Room 2. Each room has a certain amount of resources, denoted as M_1 and M_2 accordingly, and let $M = M_1 + M_2$. Both M_1 and M_2 are fixed during the repeated game. It can be seen that our system is more general than the famous minority game [7]; in the minority game [7], values of M_1 and M_2 are the same, which is only the case of unbiased resource distribution. Agents decide at each time step on which room to enter and then divide the resources in it evenly. Here, the number of agents entering Room 1 or Room 2 at a time step is marked as N_1 or N_2 respectively ($N = N_1 + N_2$). Obviously, the goal for every agent is the same, namely, to choose the room where they can obtain more resources. The values of M_1 and M_2 are unknown to all the agents. And they cannot know the global values of M_1/N_1 and M_2/N_2 at each time step in the game either, which are the amount of resources actually distributed to one agent in each room. So an agent can only evaluate his/her performance by viewing information from his/her acquaintances. In the model, every agent has a probability of k to make another agent into his/her group (for the convenience of description, the agent himself/herself is also included in the group). Hence, the agents form a directed random network. And the network is fixed during the game. At each time step, suppose in Agent i's group, there is $M_1/N_1^i \geq M_2/N_2^i$, where N_1^i or N_2^i is the number of agents in his/her group that enter Room 1 or Room 2. As a result, for Agent i, Room 1 is the winning room. However, in other agents' eyes, Room 2 may be the winning room according to information from their own groups. It is obvious that if $k = 1$, all the agents can obtain global information. As k decreases, the information an agent can get becomes less and less, compared with the global information. And particularly, $k = 0$ means Agent i's group has himself/herself inside only. So now Agent i obtains no information from the other agents. Therefore, we may say that the value of k represents the system's information concentration, $0 \leq k \leq 1$. Note that our system's framework is closer to reality than that of [114]. In [114], market states are not affected by agents' production or exchange behaviors so that an agent can even get the exact information of which state the market will be in before his/her decision is made.

Now for our agent-based model, what is left is the design of agents' decision-making process. In order to test our system under a variety of M_1/M values, we adopt the design from the market-directed resource allocation game [21] which models biased or unbiased resource distribution problems satisfactorily. In the model, every agent will create S strategies before the game start. Every strategy has two columns and a particular one is shown in Table 8. The left column is P exogenous situations (here exogenous situations mean a combination of endogenous situations, i.e., history tracks of the winning room one agent observes, and the other situations that affect one agent's decisions. It was shown in [105] that statistical properties are almost the same for models that use either exogenous situation strategies or endogenous situation ones). For every situation, the right column offers a choice respectively and here the number 1 is for the choice of Room 1 and 0 for the choice of Room 2. When an agent creates a strategy, he/she will first randomly choose a number L from 0 to P and then fill the number 1 in the right column of the strategy with a probability of L/P, and so 0 with a $(P - L)/P$ probability (here we can see the difference in strategy creation process between the market-directed resource allocation game [21] and the minority game [7]. In the minority game, the right column of a strategy is filled in by 1 or 0 with equal probabilities, i.e., both with a probability of 0.5). The strategies are fixed once they are created before the game start. At each time step, a particular exogenous situation will be picked randomly from 1 to P. Every agent will use his/her highest-scored strategy to choose rooms under the current

Table 8
A particular strategy.
Source: Adapted from Ref. [115].

Exogenous situation	Choice
1	0
2	1
3	1
.	.
.	.
.	.
P-1	0
P	1

situation. And after each time step, every agent will then assess the performance of his/her strategies based on the partial information obtained from his/her group. For example, to Agent i, if Room 1 is winning, all his/her strategies that offer the right choice at the given situation will be added one point. In the simulations, we set $S = 8$ and $P = 16$. Based on the former theoretical analysis [64], it can be calculated that now $(\langle N_1\rangle/N)_{\max} = 1 - \frac{1}{P}\sum_{\tilde{L}=1}^{P}[(\frac{\tilde{L}}{P+1})^S] = 0.911$. And for $M_1/M \le (\langle N_1\rangle/N)_{\max}$, namely, $M_1/M \le 0.911$, the system in which agents all get access to global information (i.e., $k = 1$) can reach the optimal (balanced) state where $\langle N_1\rangle/N = M_1/M$. Here $\langle\cdots\rangle$ denotes the time average of $\cdots$. Hence, we vary the value of M_1/M from 0.667 to 0.9 in the simulations.

At the beginning of a repeated game, every agent randomly creates his/her S strategies and a directed network is also linked stochastically. Then this micro-structure is fixed during the game. Therefore, in our simulations, the concept of ensemble is used to analyze the average properties of the systems that have the same parameter settings but with different micro-structures.

7.2. Controlled experiments

It is well known that for a targeted system, there can be many different designs in the agent-based modeling method. Thus, how to validate a model is becoming crucial. One way is to compare simulated results with field data [116,117,68,118–120], and the other is to do controlled experiments in laboratory systems [21,64,65,111,121]. In this research, we prefer the latter, and recruited 25 students from the Department of Physics at Fudan University as the human subjects. The game settings are the same as our agent-based model except that artificial agents are now replaced by human subjects. Two kinds of money rewards are provided to give the subjects incentives: (A) Each subject will be given one virtual point if he/she chooses the right room at each time step, and after the experiments, the accumulative virtual points a subject wins will be exchanged to cash under the ratio: 1 point = 1 Chinese Yuan; (B) Bonuses of 150, 100, and 50 Chinese Yuan are also offered to the three best-performed subjects, respectively. The rules and rewards are made clear to the students before the experiments [26]. A round of repeated pre-game with 15 time steps is also offered to the students to let them be more familiar with the rules. Each set of parameters (i.e., k and M_1/M) is then carried out to the students for one round with 15 time steps. Note that each set of k's and M_1/M's values is selected randomly before each round of repeated game so that the students can hardly figure out whether his/her experience gained in the last game can be used in the next one.

7.3. Results

The blue stars in Fig. 28 show the ensemble average of $\langle N_1\rangle/N$ versus M_1/M under four different values of k for 50 simulated systems. The error bars represent standard deviation (denoted as SD_m) among the 50 systems' $\langle N_1\rangle/N$ values. The experimental results are given in Tables 9–12. For each set of k and M_1/M, $\langle N_1\rangle/N$ (represented as red circles in Fig. 28) is calculated on the last 5 experimental time steps in order to avoid relaxation time. In both simulations and experiments, $N = 25$. Deviations between the simulated and experimental results in each sub-figure (denoted as Dev) are defined as the average of $|(\langle N_1\rangle/N)_e - (\langle N_1\rangle/N)_m|/SD_m$ over all the points (here $|\cdots|$ denotes the absolute value of $\cdots$; the subscript m stands for the simulations and e for the experiments). For the four sub-figures, the values of Dev are (a) 2.33, (b) 1.56, (c) 1.81, and (d) 2.71, respectively. Suppose in the simulated ensemble, $\langle N_1\rangle/N$ follows a Gaussian distribution, then the experimental values of $\langle N_1\rangle/N$ fall in (a) 98%, (b) 88.2%, (c) 93%, and (d) 99.4% confidence interval accordingly. In the experiments, there always exist some uncontrolled factors, such as mood swings of the students during the games. So for Fig. 28, it can be said that our agent-based model has a good mimic of the human system. Thanks to the flexibility of the agent-based modeling method, we further extend the number of agents to $N = 1001$ and the ensemble size to 500. The simulated $\langle N_1\rangle/N$ versus k is shown in Fig. 29. It can be seen that even for a small value of information concentration, e.g., $k = 0.2$, the system can still reach the optimal (balanced) state where $\langle N_1\rangle/N = M_1/M$. This means that the invisible hand can still influence the system when only a small part of information is distributed to every agent.

For the system discussed in Fig. 29, the other important property is the fluctuation level which is given by

$$Var = \frac{1}{2N}\sum_{i=1}^{2}\langle(N_i - \langle N_i\rangle)^2\rangle \equiv \frac{1}{N}\langle(N_1 - \langle N_1\rangle)^2\rangle. \tag{11}$$

Table 9
Experimental data of N_1/N's under different values of M_1/M for $k = 0.2$ within the 15 time steps. The last 5 time steps are taken to calculate the time average value of N_1/N (denoted as "Average" in the table), i.e., $\langle N_1 \rangle/N$ shown in Fig. 28.
Source: Adapted from Ref. [115].

Time step	$M_1/M = 0.667$	=0.75	=0.8	=0.833	=0.9
1	0.6	0.56	0.56	0.72	0.8
2	0.6	0.68	0.68	0.76	0.8
3	0.64	0.72	0.64	0.68	0.8
4	0.56	0.6	0.76	0.64	0.72
5	0.68	0.76	0.76	0.76	0.84
6	0.64	0.6	0.76	0.76	0.76
7	0.52	0.76	0.72	0.76	0.84
8	0.6	0.68	0.72	0.72	0.76
9	0.6	0.48	0.64	0.76	0.84
10	0.56	0.68	0.72	0.68	0.8
11	0.64	0.64	0.84	0.76	0.76
12	0.68	0.56	0.64	0.68	0.84
13	0.56	0.72	0.64	0.72	0.88
14	0.68	0.72	0.56	0.76	0.84
15	0.68	0.64	0.64	0.68	0.8
Average	0.648	0.656	0.664	0.72	0.824

Table 10
Experimental data of N_1/N's under different values of M_1/M for $k = 0.52$ within the 15 time steps. The last 5 time steps are taken to calculate the time average value of N_1/N (denoted as "Average" in the table), i.e., $\langle N_1 \rangle/N$ shown in Fig. 28.
Source: Adapted from Ref. [115].

Time step	$M_1/M = 0.667$	=0.75	=0.8	=0.833	=0.857	=0.875	=0.889	=0.9
1	0.64	0.64	0.56	0.8	0.76	0.8	0.76	0.88
2	0.6	0.68	0.72	0.48	0.76	0.64	0.76	0.6
3	0.6	0.6	0.68	0.72	0.88	0.76	0.76	0.72
4	0.68	0.68	0.8	0.76	0.68	0.84	0.84	0.88
5	0.52	0.64	0.72	0.84	0.84	0.8	0.76	0.72
6	0.72	0.84	0.72	0.52	0.72	0.64	0.84	0.84
7	0.64	0.68	0.72	0.68	0.8	0.76	0.76	0.68
8	0.44	0.88	0.72	0.84	0.84	0.84	0.76	0.8
9	0.56	0.6	0.72	0.68	0.6	0.8	0.8	0.84
10	0.68	0.8	0.72	0.76	0.72	0.76	0.76	0.76
11	0.68	0.68	0.68	0.88	0.76	0.68	0.72	0.72
12	0.56	0.68	0.72	0.84	0.8	0.76	0.84	0.84
13	0.64	0.8	0.64	0.8	0.84	0.88	0.8	0.84
14	0.68	0.76	0.76	0.84	0.88	0.84	0.84	0.92
15	0.72	0.64	0.8	0.8	0.84	0.88	0.84	0.88
Average	0.656	0.712	0.72	0.832	0.824	0.808	0.808	0.84

Table 11
Experimental data of N_1/N's under different values of M_1/M for $k = 0.76$ within the 15 time steps. The last 5 time steps are taken to calculate the time average value of N_1/N (denoted as "Average" in the table), i.e., $\langle N_1 \rangle/N$ shown in Fig. 28.
Source: Adapted from Ref. [115].

Time step	$M_1/M = 0.667$	=0.75	=0.8	=0.833	=0.857	=0.875	=0.889	=0.9
1	0.52	0.8	0.8	0.76	0.84	0.64	0.84	0.68
2	0.88	0.44	0.64	0.84	0.6	0.84	0.64	0.8
3	0.6	0.64	0.8	0.76	0.88	0.76	0.8	0.8
4	0.56	0.76	0.68	0.72	0.68	0.8	0.8	0.88
5	0.76	0.64	0.64	0.88	0.72	0.8	0.8	0.76
6	0.68	0.76	0.76	0.72	0.84	0.88	0.8	0.84
7	0.6	0.68	0.68	0.8	0.8	0.72	0.92	0.88
8	0.56	0.68	0.84	0.76	0.68	0.84	0.8	0.8
9	0.72	0.68	0.68	0.8	0.88	0.76	0.84	0.88
10	0.52	0.6	0.76	0.72	0.8	0.72	0.76	0.84
11	0.6	0.76	0.8	0.76	0.92	0.68	0.8	0.76
12	0.68	0.76	0.76	0.88	0.92	0.84	0.84	0.88
13	0.64	0.76	0.76	0.8	0.64	0.88	0.8	0.96
14	0.68	0.68	0.84	0.84	0.64	0.88	0.92	0.8
15	0.6	0.8	0.84	0.76	0.84	0.84	0.92	0.84
Average	0.64	0.752	0.8	0.808	0.792	0.824	0.856	0.848

Table 12
Experimental data of N_1/N's under different values of M_1/M for $k = 1$ within the 15 time steps. The last 5 time steps are taken to calculate the time average value of N_1/N (denoted as "Average" in the table), i.e., $\langle N_1 \rangle/N$ shown in Fig. 28.
Source: Adapted from Ref. [115].

Time step	$M_1/M = 0.667$	=0.833	=0.9
1	0.6	0.8	0.8
2	0.88	0.76	0.88
3	0.8	0.76	0.8
4	0.6	0.8	0.84
5	0.68	0.72	0.72
6	0.6	0.76	0.92
7	0.64	0.88	0.84
8	0.68	0.8	0.84
9	0.64	0.64	0.6
10	0.64	0.72	0.76
11	0.68	0.8	0.8
12	0.68	0.84	0.92
13	0.64	0.84	0.92
14	0.68	0.8	0.88
15	0.72	0.84	0.92
Average	0.68	0.824	0.888

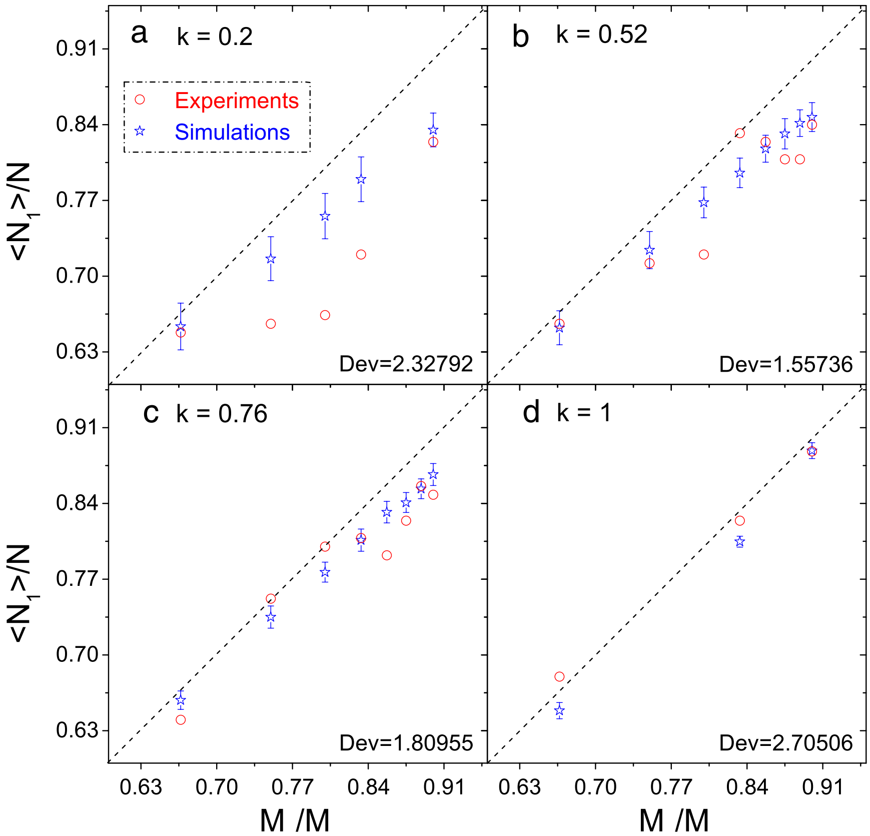

Fig. 28. $\langle N_1 \rangle/N$ versus M_1/M for $k =$ (a) 0.2, (b) 0.52, (c) 0.76, and (d) 1. The simulated ensemble contains 50 systems, each of which has the same parameters: $N = 25$, $S = 8$, and $P = 16$. Each system evolves for 600 time steps (the first half for stabilization which are enough for system relaxation and the second half for statistics). The blue stars show the ensemble average of $\langle N_1 \rangle/N$ and error bars are added. The red circles show the experimental results and for each data point, one system with the associated parameters is conducted for 15 time steps and $\langle N_1 \rangle/N$ is calculated on the last 5 time steps shown in Tables 9–12 in order to avoid the relaxationtime steps. The number of human subjects recruited in the laboratory system is the same with the simulations, i.e., $N = 25$. The diagonal dash line with slope = 1 indicates the optimal (balanced) state: $\langle N_1 \rangle/N = M_1/M$.
Source: Adapted from Ref. [115].

The ensemble average of the simulated system's fluctuation level, denoted as $Mean(Var)$, is shown in Fig. 30. It can be seen that $Mean(Var)$ declines slightly when information concentration k increases from zero. This is normal due to the following reason: as k increases, Agent i's group becomes larger, and then the information he/she obtains at every time step will be

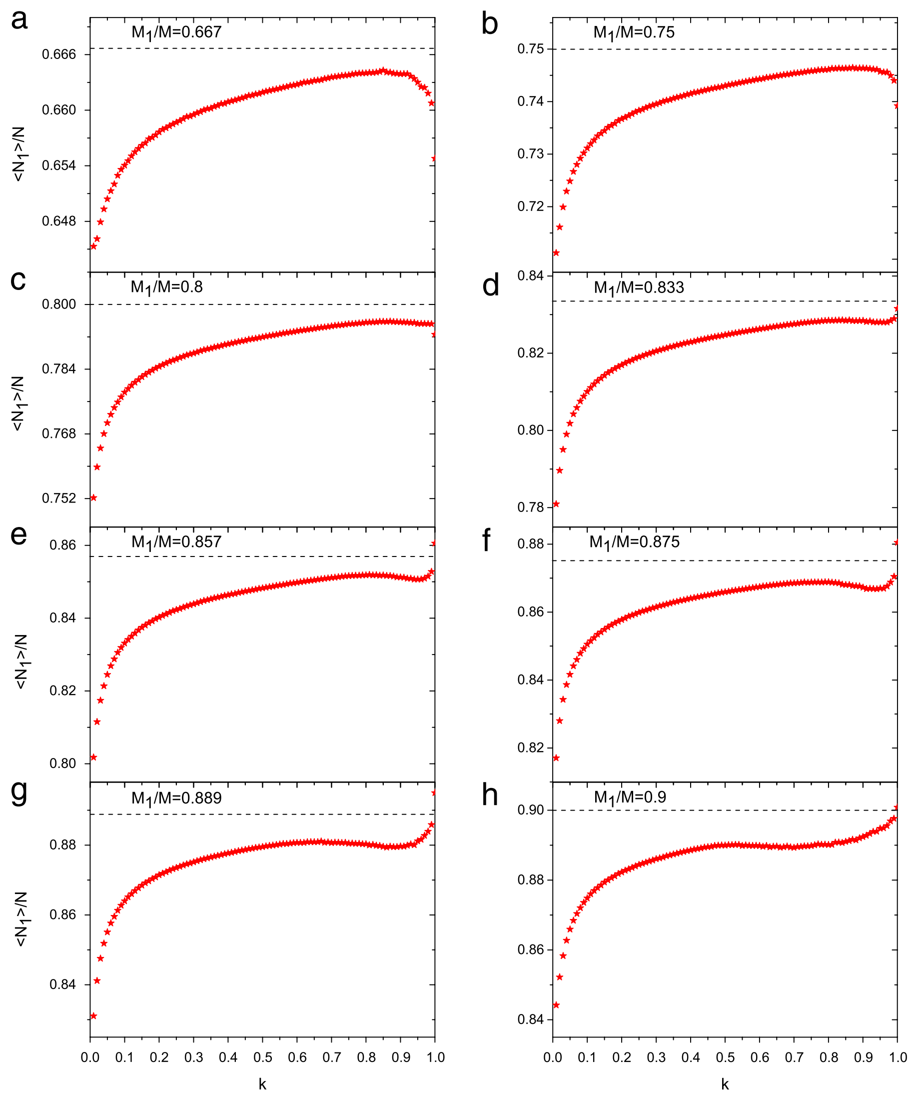

Fig. 29. Simulated results of $\langle N_1\rangle/N$ versus k for different values of M_1/M. The values of $\langle N_2\rangle/N$ are not shown here because of $\langle N_2\rangle/N = 1 - \langle N_1\rangle/N$. The ensemble contains 500 systems each of which has the same parameters: $N = 1001$, $S = 8$, and $P = 16$. Each system evolves for 600 time steps (the first half for stabilization which are enough for system relaxation and the second half for statistics). The horizontal dash line shows the value of $\langle N_1\rangle/N$ for the optimal (balanced) state: $\langle N_1\rangle/N = M_1/M$.
Source: Adapted from Ref. [115].

more stable, which makes Agent i more certain about his/her choices. Hence, we interpret it as a normal phase. But after that, $Mean(Var)$ climbs up abnormally, which means that now agents getting more information will be more harmful to the system's stability (a higher fluctuation level means a lower stability of the system); this is in contrast to the belief that more information is better, so we interpret this as an abnormal phase. This phenomenon can be explained by the following feedback process. For $k = 1$, because of the full network connection, uncertainties in Agent i's choice can be transferred to all the other agents and increase their own uncertainties, and then the other agents' uncertainties will be again transferred back to Agent i and further increase his/her choice fluctuations. However, when k decreases from 1, owing to the presence of the directed network, for one acquaintance in Agent i's group, he/she may not have Agent i in his/her own group. This means the uncertainties of Agent i cannot be fully transferred to his/her group members now, so the overall fluctuation level will decrease. Hence, here a continuous phase transition between a normal phase and an abnormal phase comes to appear as k increases from 0 to 1. The lowest $Mean(Var)$ point is labeled as the transition point and the associated critical value of k is denoted as k_c. Detailed information around the transition point is shown in the insets of Fig. 30.

In the traditional continuous phase transition theory [35], it is stated that around a transition point, fluctuations will increase heavily since the correlation length becomes infinite. On the contrary, in Fig. 30, it can be seen that at the transition point, the ensemble fluctuations of the fluctuation level, denoted as $Var(Var)$, remain at a low value, which is only of

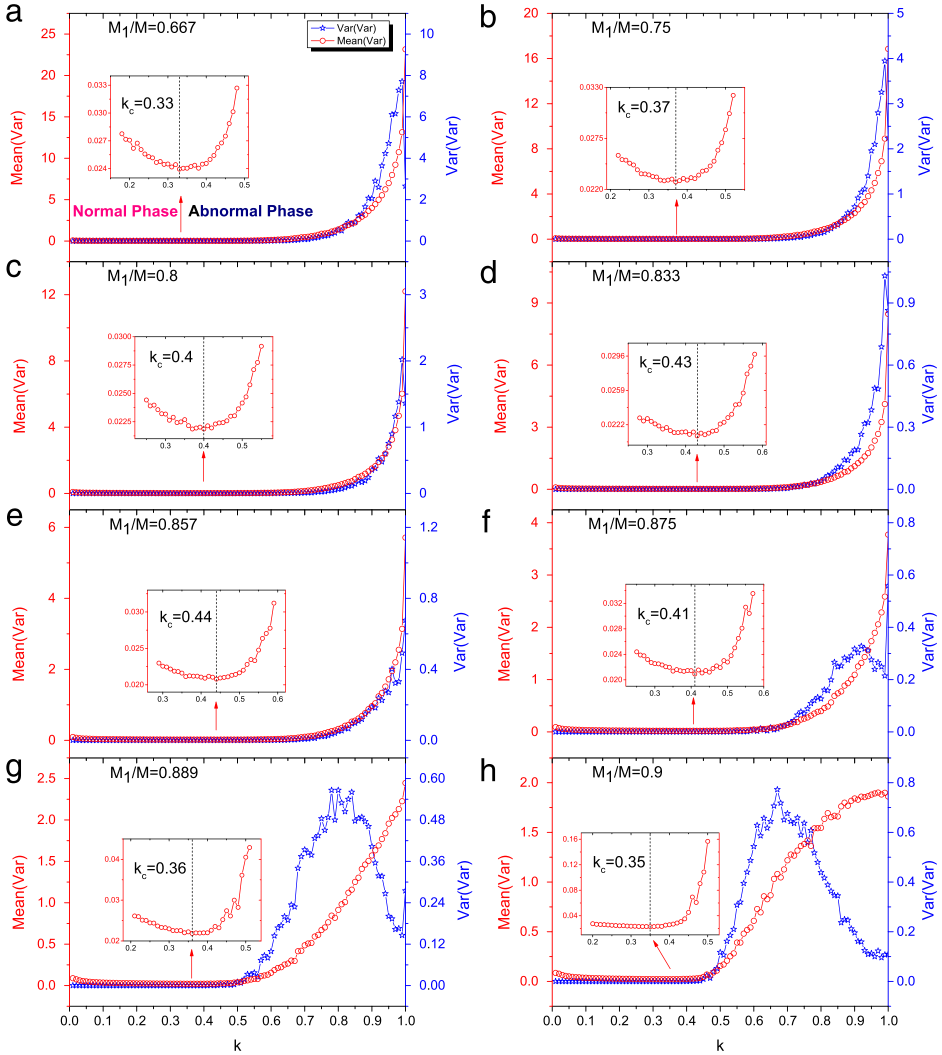

Fig. 30. Fluctuations of the simulated system. For the eight M_1/M values, the ensemble averages of the fluctuation level (denoted as $Mean(Var)$) under different k values are represented by red circles, while blue stars are for the ensemble fluctuations of the fluctuation level (denoted as $Var(Var)$). $Mean(Var)$ goes through a continuous phase transition from a normal phase to an abnormal phase as k increases. The insets give detailed information around the transition point (k_c stands for the critical value of k) accordingly. The ensemble contains 500 systems, each of which has the same parameters: $N = 1001$, $S = 8$, and $P = 16$. Each system evolves for 600 time steps (the first half for stabilization which are enough for system relaxation and the second half for statistics).
Source: Adapted from Ref. [115].

magnitude 10^{-5} versus $Mean(Var)$ being 10^{-2}. However, at the abnormal phase where k is large, $Var(Var)$ has a great rise as k increases, and for $M_1/M \geq 0.875$, $Var(Var)$ even begins to decline obviously when k increases further.

The above phase transition phenomenon comes to appear in our system that contains a limited number of agents. So we attempt to analyze the relation between k_c and N. Because the critical information concentration k_c shows no relation with M_1/M in Fig. 30, we average it over different values of M_1/M and obtain $\overline{k}_c$. Fig. 31 displays a power-law relation between $\overline{k}_c$ and N in the tail (i.e., for $N \geq 401$) as $\overline{k}_c = 50 * N^{-0.71}$. Hence, when the system becomes infinitely large, i.e., $N \to +\infty$, there still exists this anomalous phase transition at $\overline{k}_c \to 0^+$.

7.4. Concluding remarks

In Section 7, we have designed an agent-based model with partial information for biased resource-allocation problems. A series of controlled human experiments have been conducted to show the reliability of the model design. We have found that even for a small information concentration, the system can still reach the optimal (balanced) state. Furthermore, we have found that the ensemble average of the simulated system's fluctuation level has a continuous phase transition. This means that in the abnormal phase, too much information can hurt the system's stability instead. Hence, back to the question

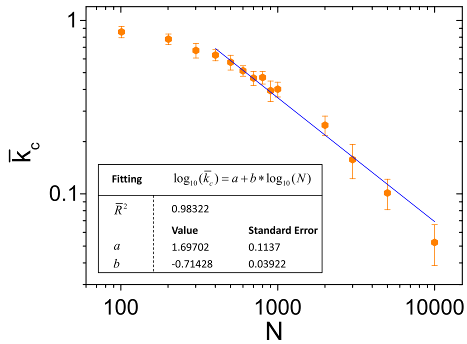

Fig. 31. A log–log plot showing $\bar{k}_c$ versus N. Here, $\bar{k}_c$ is the average of k_c's under different values of M_1/M. For each value of N, the ensemble contains 50 systems each of which has the same parameters: $S = 8$ and $P = 16$. Each system evolves for 600 time steps (the first half for stabilization which are enough for system relaxation and the second half for statistics). The blue line fits the tail of the data (i.e., from $N = 401$ to $N = 10,001$) according to $\log_{10}(\bar{k}_c) = a + b * \log_{10}(N)$; details are given in the inset where the regression coefficient, $\bar{R}^2 = 0.98322$, indicates well fitting of the tail since $\bar{R}^2 = 1.0$ means a perfect fit [122]. Error bars are added as well.
Source: Adapted from Ref. [115].

raised at the beginning, we can say that markets are able to use the invisible hand most efficiently only at the transition point where partial information is obtained by agents.

On the other hand, at the transition point, the ensemble fluctuations of the fluctuation level remain at a low value. When increasing the number of agents, the critical value of information concentration obeys a power-law decay in the tail. This behavior confirms that when the system becomes infinitely large, there still exists this kind of fluctuation transition phenomena. This finding is in contrast to the textbook knowledge about continuous phase transitions also addressed at the beginning, namely, fluctuations will rise abnormally around a transition point since the correlation length becomes infinite. This may pave the way for investigating the role of human adaptability in further developing traditional physics.

Therefore, Section 7 is expected to be of value to some different fields ranging from physics, to economics, to complexity science, and to artificial intelligence.

8. Risk management

Subjects in all kinds of laboratory markets, introduced in Sections 3–7, are always facing risks. In Section 8, now I am in a position to introduce the relationship between risk and return.

For survival and development, agents in various kinds of complex adaptive systems (CASs) involving human society must compete against or collaborate with each other for sharing limited resources or wealth, by utilizing different methods. One of the methods is to invest, in order to obtain payoffs with risk. Accordingly, understanding the risk–return relationship (RRR) is of both academic value and practical importance. So far this relationship has a two-fold character. On one hand, investments are considered as high risk high return and vice versa; the RRR is positive (risk–return tradeoff) [123,124]. This is also an outcome of the traditional financial theory under the efficient market hypothesis. On the other hand, some investments are found high risk low return and vice versa; the RRR is negative (Bowman's paradox) [125,126]. However, almost all investment products take "high risk high return" as a bright spot to attract investors, and neglect the possible existence of "high risk low return". This actually results from a received belief that investments with a positive RRR are dominant over those with a negative RRR in the human society; the belief directs investors to operate investing activities including gambling [127]. Here we investigate the RRR by designing and investigating a model CAS which includes the following two crucial factors:

- *Market efficiency.* The present system exhibits market efficiency at which it reaches a statistical equilibrium [64,21]. We shall address more relevant details at the end of the next section.

- *Closeness.* The system involves two conservations: one is the population of investors (Conservation I), the other is wealth (Conservation II). Regarding Conservation I/II, we fix the total number/amount of the subjects/wealth in the system.

Clearly the two factors have real traces in human society. Accordingly they have played an important role in helping to establish traditional finance/economics theories. The present designing system just allows us to investigate the joint effect of the two factors on the RRR.

8.1. Controlled experiments

On the basis of the CAS, we conducted a series of computer-aided human experiments. Details are as follows. There are two virtual rooms, Room 1 and Room 2 (represented by two buttons on the computer screen of the subjects), for subjects to invest in. The two rooms have volumes M_1 and M_2, which may represent the arbitrage space for a certain investment in the real world. For the experiments, we recruited 24 students from Fudan University as subjects. These subjects acted as fund managers, who were responsible for implementing investing strategy of the fund and managing its trading activities.

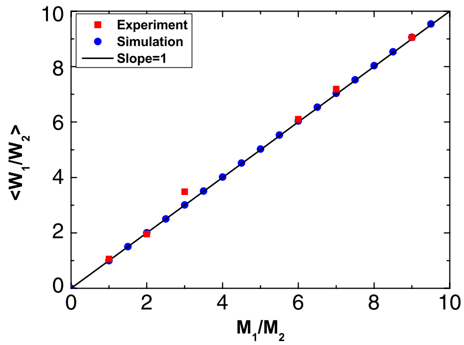

Fig. 32. Averaged ratio, $\langle \mathbf{W}_1/\mathbf{W}_2\rangle$, versus $\mathbf{M}_1/\mathbf{M}_2$ for the human experiments with 24 subjects (red squares) and agent-based computer simulations with 1000 agents (blue dots). Here "$\langle\cdots\rangle$" denotes the average over the total 30 experimental rounds (experimental data of W_1/W_2 for each round are shown in Table 13) or over the 800 simulation rounds (the additional 200 rounds were performed at the beginning of the simulation for each M1/M2; during the 200 rounds, we train all of the strategies by scoring them whereas the wealth of each agent remains unchanged). All the experimental and simulation points lie on or beside the diagonal line ("slope = 1"), which is indicative of $\langle W_1/W_2\rangle \approx M_1/M_2$. Parameters for the simulations: $S = 4$ and $P = 16$.
Source: Adapted from Ref. [111].

We told the subjects the requirement of total 30 rounds for every single M_1/M_2, and offered every subject 1000 points (the amount of virtual money constructs the fund managed by the subject) as his/her initial wealth for each M_1/M_2. In an attempt to make the subjects maximize their pursuit of self-interest, we promised to pay the subjects Chinese Yuan according to a fixed exchange rate, 100:1 (namely, one hundred points equal to one Chinese Yuan), at the end of the experiments, and to offer every subject 30 Chinese Yuan as a bonus of attendance. Extra 50 Chinese Yuan would be given to the subject who gets the highest score for a single M_1/M_2. At the beginning of the 1st round of each M_1/M_2, we told the 24 subjects the value of M_1/M_2, and asked each subject to decide his/her investing weight [signed as $x(i)$ for Subject i]. Note the investing weight, $x(i)$, is the percentage of his/her investing wealth (investment capital) with respect to his/her total wealth, and it will keep fixed within the 30 rounds for a certain M_1/M_2. In each round, each subject can only invest in one of the two rooms independently. After all the subjects made their decisions, with the help of the computer program, we immediately knew the total investments in each room (signed as W_1 and W_2 for Room 1 and Room 2, respectively) in this round. While keeping the total wealth conserved, we redistributed the total investment $W_1 + W_2$ according to the following two rules:

(1) We divided the total investment, W_1+W_2, by the ratio of M_1 and M_2, yielding $(W_1+W_2)\frac{M_1}{M_1+M_2}$ and $(W_1+W_2)\frac{M_2}{M_1+M_2}$ as the payoff for Room 1 and Room 2, respectively.

(2) We redistributed the payoff of Room k ($k = 1$ *or* 2) by the investment of the subjects. Namely, for each round, the payoff for Subject i choosing Room k to invest in, $w_{payoff}(i)$, is determined by $w_{payoff}(i) = (W_1 + W_2)\frac{M_k}{M_1+M_2} \times \frac{w_{in}(i)}{W_k}$, where $w_{in}(i)$ is the investing wealth of Subject i, $w_{in}(i) = x(i)w(i)$. Here $w(i)$ is the total wealth possessed by Subject i at the end of the previous round.

Before the experiments, we told the subjects the above two rules for wealth re-allocation. After each round, every subject knows his/her payoff, $w_{payoff}(i)$. If there is $w_{payoff}(i) > w_{in}(i)$, that is, Subject i gets more than the amount he/she has invested, we consider Subject i as a winner at this round. Equivalently, if $\frac{W_1}{M_1} < \frac{W_2}{M_2}$, the subjects choosing Room 1 to invest in win. Clearly, when $\frac{W_1}{W_2} = \frac{M_1}{M_2}$, every subject obtains the payoff which equals to his/her investing wealth. Namely, the arbitrage opportunity has been used up. Accordingly, we define the $\frac{W_1}{W_2} = \frac{M_1}{M_2}$ state as an equilibrium (or balanced) state [7]. This state may have some practical significance because global arbitrage opportunities for investing in the human society always tend to shrink or even disappear once known and used by more and more investors. As shown in Fig. 32 (as well as Table 13), our experimental system can indeed achieve $\langle W_1/W_2\rangle \approx M_1/M_2$ at which the system automatically produces the balanced allocation of investing wealth; this system thus reaches a statistical equilibrium. In other words, the "Invisible Hand" plays a full role [21], or alternatively the system exhibits market efficiency. That is, all subjects are pursuing self-interest and we run the present system under three conditions: with sufficient information (namely, the wealth change for each round reflects the possible information), with free competition (i.e., no subjects dominate the system and there are zero transaction costs), and without externalities (the wealth change of a subject reflects the influence of his/her behavior on the others).

If a subject chooses a large investing weight, he/she will invest more virtual money in a room. According to the rules of our experiment, the room he/she chooses will then be more likely to be the losing one. Besides, the initial wealth is the same for every subject and he/she knows nothing about the others. From this point of view, the larger investing weight he/she chooses, the higher risk (or uncertainty) he/she will take for the fund (i.e., the initial 1000 points). Therefore, throughout Section 8, we simply set the investing weight, $x(i)$, to equal the risk he/she is willing to take. Here we should remark that the present definition of risk appears to be different from that in financial theory. For the latter, one often defines risk according to variance. Nevertheless, these two "risk"s are essentially the same because they both describe the uncertainty of funds and have a positive association with each other. On the other hand, we should mention that the risk for each subject does not change with the evolution of time. This is a simplification which makes it possible to discuss the pure effect of a fixed value

Table 13
Experimental data of W_1/W_2's for six M_1/M_2's within 30 rounds.
Source: Adapted from Ref. [111].

Round	$M_1/M_2 = 1$	=2	=3	=6	=7	=9
1	1.247723	1.143654	5.267782	2.98977	24.41429	2.146853
2	0.582237	0.702725	1.717598	11.02642	6.827457	4.860541
3	0.759914	1.897306	2.43237	10.32266	11.25343	11.30546
4	1.903253	1.240914	2.699907	2.97036	5.688661	9.681926
5	1.940527	1.564242	3.999681	3.977399	6.546176	5.249869
6	1.4852	4.711605	2.815152	6.900399	5.13295	6.16301
7	0.71966	2.087147	8.280381	2.991117	9.27272	7.25918
8	0.675138	1.692307	4.590899	3.35285	7.12301	8.996662
9	1.029128	2.73341	1.833477	4.363129	4.329496	7.133701
10	0.867554	2.095702	3.063358	7.273544	8.198398	14.26918
11	1.50125	1.305197	3.862686	18.23372	5.927536	5.500789
12	0.846259	2.292878	3.826587	8.50234	4.673143	5.141253
13	0.629585	1.992493	5.31337	4.613084	13.47519	34.4646
14	0.784858	2.462247	4.687499	19.73941	4.867279	3.889573
15	1.484235	1.807911	2.991726	3.40541	9.820732	7.442826
16	2.309969	1.544355	3.301258	4.864645	19.63957	15.74645
17	1.01251	2.078769	1.009523	8.219743	4.389477	11.55617
18	0.987891	2.624829	1.531467	2.935522	6.684373	8.712361
19	1.319123	2.25104	2.29988	3.813827	6.655679	6.623739
20	0.872338	2.045779	3.140856	5.690231	9.253236	7.973963
21	1.166773	2.006077	5.282071	5.889009	5.021116	5.825073
22	0.896165	1.419159	3.53215	6.137386	7.409623	8.32772
23	0.872224	2.141954	2.629218	11.09127	7.033376	15.57089
24	1.275063	1.990766	4.722947	5.989491	7.216511	10.87512
25	0.695696	2.151347	3.410795	7.790409	8.787551	4.759215
26	1.149307	2.150258	3.400615	8.213546	6.472158	13.14246
27	1.379602	1.621164	5.898509	5.078065	6.915495	7.992252
28	0.809361	1.62651	2.421057	3.698009	5.514453	11.76899
29	0.772988	1.670855	3.576442	7.848631	7.483899	16.27463
30	0.367173	2.010509	2.90843	11.10609	8.9996	4.854004

Table 14
Linear fitting functions for Fig. 33(a)–(l).
Source: Adapted from Ref. [111].

M_1/M_2	For the experimental data	For the simulation data
1	$r_T(i) = 0.17 - 0.31x(i)$ [Fig. 33(a)]	$r_T(i) = 0.40 - 0.82x(i)$ [Fig. 33(g)]
2	$r_T(i) = 0.0073 - 0.036x(i)$ [Fig. 33(b)]	$r_T(i) = 0.53 - 1.08x(i)$ [Fig. 33(h)]
3	$r_T(i) = 0.49 - 0.74x(i)$ [Fig. 33(c)]	$r_T(i) = 0.37 - 0.76x(i)$ [Fig. 33(i)]
6	$r_T(i) = 0.31 - 0.41x(i)$ [Fig. 33(d)]	$r_T(i) = 0.44 - 0.89x(i)$ [Fig. 33(j)]
7	$r_T(i) = 0.24 - 0.29x(i)$ [Fig. 33(e)]	$r_T(i) = 0.35 - 0.68x(i)$ [Fig. 33(k)]
9	$r_T(i) = 0.26 - 0.33x(i)$ [Fig. 33(f)]	$r_T(i) = 0.20 - 0.38x(i)$ [Fig. 33(l)]

of "risk". Nevertheless, if we choose to let the "risk" change with the time, for the same purpose, we may take an average of the "risk" over the full range of time. Fig. 33(a)–(f) displays the risk–return relationship for the investments in the designing CAS. From statistical point of view, we find that investments with a negative RRR are dominant over those with a positive RRR in the whole system.

8.2. Agent-based modeling

Obviously the human experiments have some unavoidable limitations: specific time, specific avenue (a computer room of Fudan University), specific subjects (students from Fudan University), and the limited number of subjects. Now we are obliged to extend the experimental results [Fig. 33(a)–(f)] beyond such limitations. For this purpose, we resort to an agent-based model [7,79,95].

Similar to the above experiments, we set two virtual rooms, Room 1 and Room 2 (with volume M_1 and M_2, respectively), for N agents (fund managers) to invest in. Then, for each M_1/M_2, assign every agent 1000 points as his/her initial wealth and an investing weight, $x(i)$, which is randomly picked up between 0 and 1 with a step size of 0.001. In order to avoid the crowding or overlapping of strategies of different agents [128,129,81], we design the decision-making process for each agent with four steps.

–*Step* 1: set a positive integer, P, to represent the various situations for investing [64,21].

–*Step* 2: assign each agent S strategies according to S integers between 0 and P, respectively. For example, if one of the S integers is L, then the corresponding strategy of the agent is given by the ratio $L/P(0 \le L/P \le 1)$, which represents the probability for the agent to choose Room 1 to invest in [64].

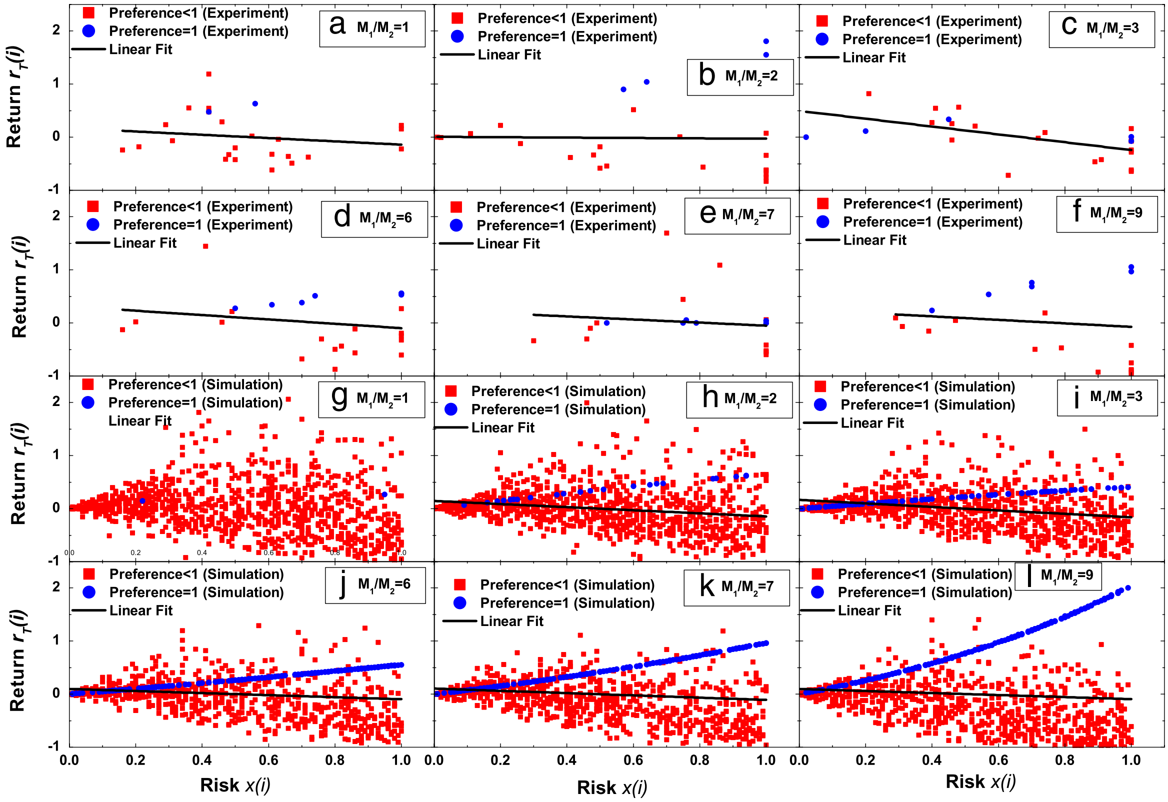

Fig. 33. Relationship between the risk, $\mathbf{x(i)}$, and the return, $\mathbf{r_T(i)} = [\mathbf{w_T(i)} - \mathbf{w_0(i)}]/\mathbf{w_0(i)}$, for (a)–(f) 24 subjects and (g)–(l) 1000 agents at various $\mathbf{M_1/M_2}$'s. (a)–(f) Data of the human experiments (total 30 rounds for each M_1/M_2); (g)–(l) Data of the agent-based computer simulations (total 800 rounds for each M_1/M_2, with additional 200 rounds performed at the beginning of the simulations; during the 200 rounds, we train all of the strategies by scoring them whereas the wealth of each agent remains unchanged). Here $w_T(i)$ is Agent i's wealth at the end of T rounds (the total number of rounds, T, is $T = 30$ *and* 800 for the experiments and simulations, respectively), and $w_0(i)$ is Agent i's initial wealth. All of the subjects or agents are divided into two groups with preference < 1 (red squares) and preference $=1$ (blue dots). Here, the "preference" is given by C_1/T, where C_1 is the number of times for subjects or agents to choose Room 1 within the total T rounds. The values or distribution of the preferences of the subjects or agents can be found in Figs. 35–36. Here, "Linear Fit" denotes the straight line fitting of the data in each panel using the least square method, which serves as a guide for the eye. (The fitting functions are listed in Table 14.) All of the lines are downward, which indicate a statistically negative relationship between risk and return. The present negative relationship just reflects the dominance of investments with a negative RRR in the whole system, in spite of a relatively small number of investments with a positive RRR. Other parameters: (g)–(l) $S = 4$ and $P = 16$.
Source: Adapted from Ref. [111].

–*Step* 3: for an agent, each strategy has its own score with an initial score, 0, and is added one score (or zero score) if the strategy predicts (or does not predict) the winning room correctly after each round.

–*Step* 4: every agent chooses either Room 1 or Room 2 to invest in according to the prediction made by the strategy with the highest score.

In addition, both the payoff function and the rules for re-distributing investing wealth in Room 1 and Room 2 are set to be the same as those already mentioned in Section 8.1.

8.3. Comparison between experimental and simulation results

As shown by Fig. 32, our agent-based computer simulations also give $\langle W_1/W_2 \rangle \approx M_1/M_2$, that is, the system under simulation also exhibits market efficiency. Furthermore, according to the simulations, we achieve the same qualitative conclusion: investments with a negative RRR are statistically dominant over those with a positive RRR in the whole system; see Fig. 33(g)–(l). Nevertheless, when we scrutinize Fig. 33(j)–(l), we find that some particular data seem to be located on a smooth upward line. We plot these data in blue, and further find that they just correspond to all the agents with "preference = 1". Encouraging by this finding, we blue all the data of "preference = 1" in the other 9 panels of Fig. 33, and observe that a similar upward line also appears in the experimental results [see the blue dots in Fig. 33(a)–(f); note the blue dots in Fig. 33(c) and (e) are also, on average, in an upward line even though they appear to be not so evident].

For the upward lines themselves, they are clearly indicative of investments with a positive RRR. Hence, to distinctly understand our main conclusion about the dominance of investments with a negative RRR in the whole system, we have to overcome the puzzle, namely, the strange appearance of these upward lines (constructed by the blue dots in Fig. 33).

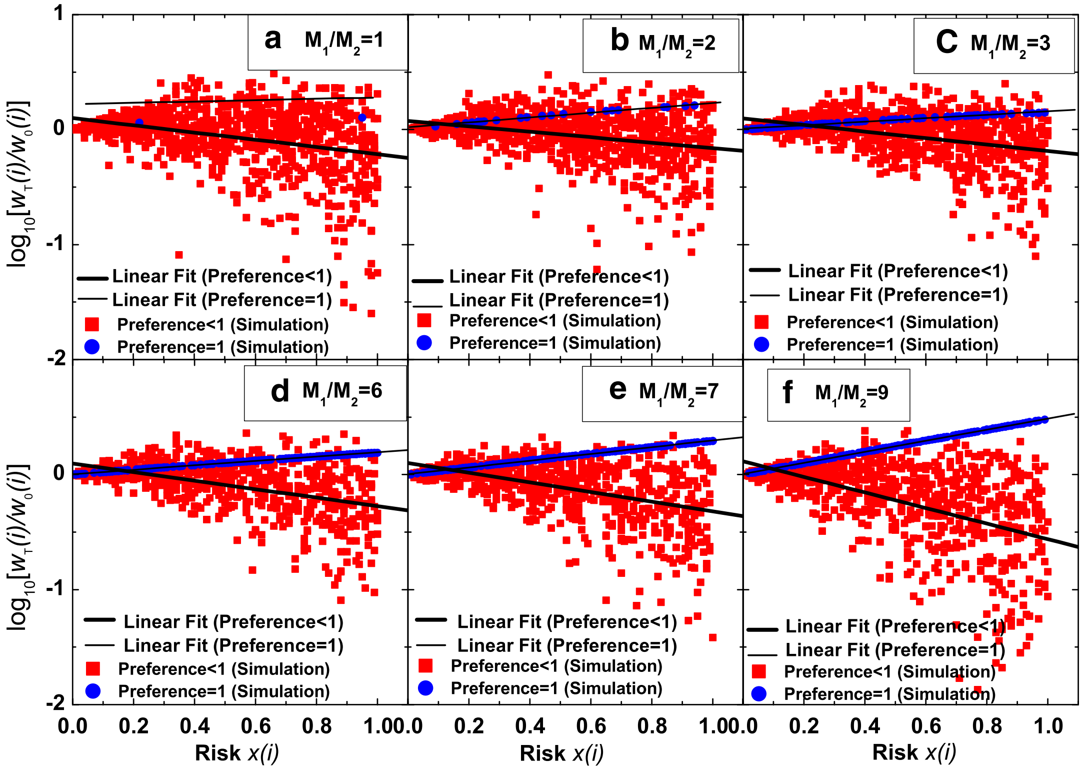

Fig. 34. Same as Fig. 33(g)–(l), but showing the relationship between the risk, $\mathbf{x(i)}$, and the relative wealth, $\mathbf{w_T(i)/w_0(i)}$, on a logarithmic scale. "Linear Fit" corresponds to the line fitting the data of preference $<$ 1 or preference $=$ 1 using the least square method, which serves as a guide for the eye. (The fitting functions are listed in Table 15.)
Source: Adapted from Ref. [111].

Table 15
Linear fitting functions for Fig. 34(a)–(l).
Source: Adapted from Ref. [111].

$\frac{M_1}{M_2}$	For "preference $<$ 1"	For "preference $=$ 1"
1	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.10 - 0.31x(i)$ [Fig. 34(a)]	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.05 + 0.058x(i)$ [Fig. 34(a)]
2	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.07 - 0.23x(i)$ [Fig. 34(b)]	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.02 + 0.24x(i)$ [Fig. 34(b)]
3	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.09 - 0.28x(i)$ [Fig. 34(c)]	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.01 + 0.05x(i)$ [Fig. 34(c)]
6	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.09 - 0.37x(i)$ [Fig. 34(d)]	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.01 + 0.19x(i)$ [Fig. 34(d)]
7	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.10 - 0.42x(i)$ [Fig. 34(e)]	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.003 + 0.29x(i)$ [Fig. 34(e)]
9	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.11 - 0.68x(i)$ [Fig. 34(f)]	$\log_{10}\frac{w_T(i)}{w_0(i)} = 0.004 + 0.48x(i)$ [Fig. 34(f)]

For convenience, we just need to answer Question 1: why do all the "preference $=$ 1" data dots of Fig. 33(g)–(l) exist in an upward line? To this end, the answer to Question 1 will also help to reveal the mechanism underlying the above main conclusion.

8.4. Comparison among experimental, simulation, and theoretical results

To answer Question 1, we attempt to study the relationship between risk and wealth; see Fig. 34. In Fig. 34, the "preference $=$ 1" data dots appear to be arranged in an upward straight line, and the straight line exactly corresponds to the upward line constructed by the blue dots in Fig. 33(g)–(l) due to the relationship between the wealth and return. So, Question 1 equivalently becomes Question 2: why do all the "preference $=$ 1" data dots of Fig. 34 exist in an upward straight line? To answer it, we start by considering Agent i with investment weight, $x(i)$. His/her return and wealth after t rounds are, respectively, $r'_t(i)$ and $w_t(i)$. Here, the subscript $t\epsilon[0, T]$. (Note T stands for the total number of simulation rounds, $T = 800$.) Clearly, when $t = 0$, $w_t(i) = w_0(i)$, which just denotes the initial wealth of Agent i. Then, we obtain the expression for $r'_t(i) = [w_t(i) - w_{t-1}(i)]/[w_{t-1}(i)x(i)]$. Accordingly, we have $w_1(i) = w_0(i)[1 + r'_1(i)x(i)]$ and $w_2(i) = w_1(i)[1 + r'_2(i)x(i)] = w_0(i)[1 + r'_1(i)x(i)][1 + r'_2(i)x(i)]$, thus yielding $w_T(i) = w_0(i)[1 + r'_1(i)x(i)] \ldots [1 + r'_T(i)x(i)] = w_0(i)\prod_{t=1}^{T}[1 + r'_t(i)x(i)]$. As

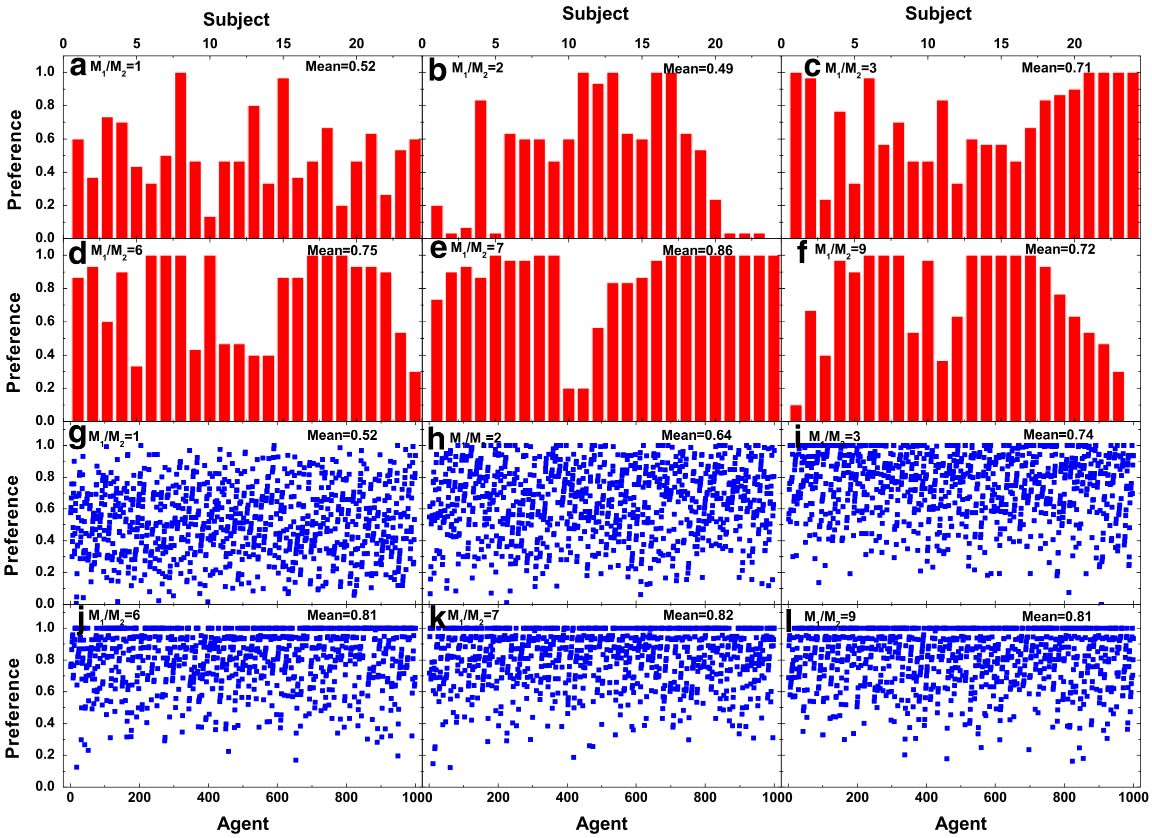

Fig. 35. Preferences of (a)–(f) the 24 subjects in the human experiments (plotted in the bar graph) or (g)–(l) the 1000 agents in the agent-based computer simulations, for various M_1/M_2's. Here, "Mean" denotes the preference value averaged for (a)–(f) the 24 subjects or (g)–(l) 1000 agents. In (a)–(f), the present 24 subjects are ranked by their risk (namely, their investing weight) from low to high, within the range (a) [0.16, 1], (b) [0.01, 1], (c) [0.02, 1], (d) [0.16, 1], (e) [0.31, 1], and (f) [0.29, 1]; see Table 16 for details. Similarly, in (g)–(l), the 1000 agents are ranked by their risk from low to high, within the range (0, 1] assigned according to the code "(double)rand()%1001/1000" in the C programming language. In (a)–(f), the ratio between the numbers of subjects with "preference = 1" and "preference < 1" are, respectively, (a) 2/22, (b) 4/20, (c) 5/19, (d) 7/17, (e) 11/13, and (f) 8/16. In (g)–(l), the ratio between the numbers of agents with "preference = 1" and "preference < 1" are, respectively, (g) 2/998, (h) 23/977, (i) 94/906, (j) 233/767, (k) 200/800, and (l) 220/780.
Source: Adapted from Ref. [111].

a result, we obtain $\log_{10}\frac{w_T(i)}{w_0(i)} = \log_{10}\{\prod_{t=1}^{T}[1+r'_t(i)x(i)]\} = \sum_{t=1}^{T}\log_{10}[1+r'_t(i)x(i)] = \left[\sum_{t=1}^{T} r'_t(i)\right]x(i) = T\langle r'_t(i)\rangle x(i)$. Here the third "=" holds due to $r'_T(i)x(i) \to 0$ for the T simulation rounds of our interest. In this equation, $\langle r'_T(i)\rangle$ denotes the average return, namely, the value obtained by averaging $r'_T(i)$ over the T rounds, and $\frac{w_T(i)}{w_0(i)}$ represents the relative wealth. Thus, the relationship between $\log_{10}\frac{w_T(i)}{w_0(i)}$ and $x(i)$ should be linear; the sign of the slope of the straight lines is only dependent on the average return, $\langle r'_T(i)\rangle$. Because the agents with preference $= 1$ always enter Room 1 with $M_1(> M_2)$, the average return, $\langle r'_T(i)\rangle$, for them is not only positive but also the same. This is why all the blue points in Fig. 34 lie on an upward straight line. However, for the other agents with preference < 1 (Fig. 34), they will change rooms from time to time, so their average return, $\langle r'_T(i)\rangle$, is different from one another. This is the reason why the red points do not form a straight line as the blue points do. From this point of view, the downward straight line we draw from the red points in Fig. 34 is just a statistical analysis showing a trend. So, the answer to Question 2 can simply be "because for the small number of agents with preference $= 1$, their average return, $\langle r'_T(i)\rangle$, is not only positive but also the same".

According to the above theoretical analysis, we can now understand that the statistical dominance of investments with a negative RRR in the whole system results from the distribution of subjects'/agents' preferences: the heterogeneous preferences (< 1) owned by a large number of subjects/agents together with the identical preferences ($= 1$) possessed by a small number of subjects/agents. Details about the actual values for the preferences can be found in Figs. 35–36. Figs. 35–36 also show the environmental adaptability of subjects or agents.

8.5. Concluding remarks

On the basis of the designed CAS (complex adaptive system), we have revisited the relationship between risk and return under the influence of market efficiency and closeness by conducting human experiments, agent-based simulations, and

Table 16
Values for the risk (namely, investing weight) of the 24 subjects for six M_1/M_2's in the human experiments. We ranked the 24 subjects by their risk from low to high, as already used in Fig. 35(a)–(f).
Source: Adapted from Ref. [111].

Subject	$\frac{M_1}{M_2} = 1$ [(a)]	=2 [(b)]	=3 [(c)]	=6 [(d)]	=7 [(e)]	=9 [(f)]
1	0.16	0.01	0.02	0.16	0.31	0.29
2	0.21	0.02	0.2	0.2	0.46	0.31
3	0.29	0.11	0.21	0.41	0.47	0.39
4	0.31	0.2	0.4	0.46	0.49	0.4
5	0.36	0.26	0.41	0.49	0.52	0.47
6	0.42	0.41	0.45	0.5	0.7	0.57
7	0.42	0.48	0.46	0.61	0.75	0.7
8	0.42	0.5	0.46	0.7	0.75	0.7
9	0.46	0.5	0.48	0.7	0.79	0.71
10	0.47	0.52	0.5	0.74	0.86	0.74
11	0.48	0.57	0.63	0.76	1.	0.79
12	0.5	0.6	0.72	0.8	1.	0.9
13	0.5	0.64	0.74	0.8	1.	1.
14	0.55	0.74	0.89	0.82	1.	1.
15	0.56	0.81	0.91	0.86	1.	1.
16	0.61	1.	1.	0.86	1.	1.
17	0.61	1.	1.	1.	1.	1.
18	0.63	1.	1.	1.	1.	1.
19	0.66	1.	1.	1.	1.	1.
20	0.67	1.	1.	1.	1.	1.
21	0.72	1.	1.	1.	1.	1.
22	1.	1.	1.	1.	1.	1.
23	1.	1.	1.	1.	1.	1.
24	1.	1.	1.	1.	1.	1.

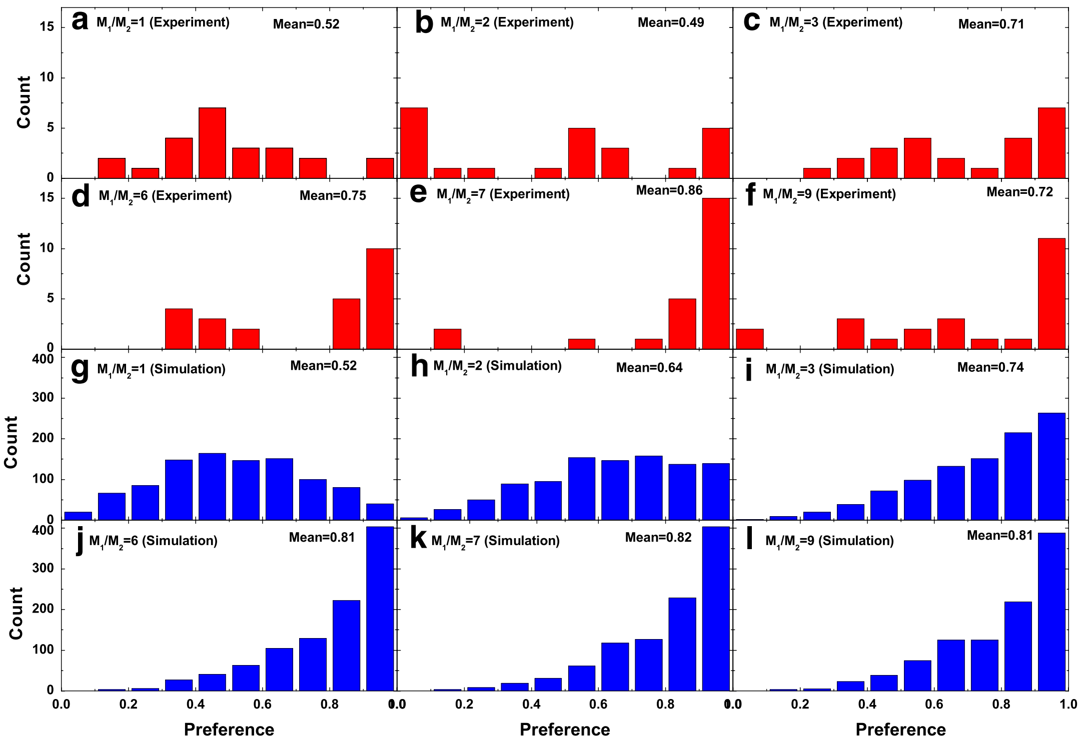

Fig. 36. Same as Fig. 35, but showing the distribution of preferences.
Source: Adapted from Ref. [111].

theoretical analysis. We have reported that investments with a negative RRR (risk–return relationship) have dominance over those with a positive RRR in this CAS. We have also revealed the underlying mechanism related to the distribution of preferences. Our results obtained for the overall system do not depend on the evolutionary time, T, as long as T is large enough. On the other hand, the experimental data for each T have been listed in Table 13. Clearly, the results for each T

can change accordingly. In fact, such changes echo with those fluctuations or volatilities yielding arbitrage opportunities for investors in the real human society.

Section 8 should be valuable not only to complexity science, but also to finance and economics, to management and social science, and to physics. In finance and economics, it may remind investors about their investing activities. In management and social science, our results are valuable to clarify the relationship between risk and return under some conditions. In physics, Section 8 helps to reveal a new macroscopic equilibrium state in such a CAS.

9. Summary and prospect

In this review, I have surveyed the field of experimental econophysics with a focus on the following topics related to economics or finance: stylized facts, herd behavior, contrarian behavior, spontaneous cooperation, partial information, and risk management. This review covers the basic concepts, methods, and latest progress in the field of experimental econophysics.

The combination of empirical analysis, controlled experiments and theoretical analysis is the fundamental method in the field of experimental econophysics (Section 1). Clearly, controlled experiments are the most important part within this combination. Thus, how to develop experimental econophysics mainly relies on how to design and perform controlled experiments convincingly. The laboratory markets presented in this review article offer an approximation of real markets, which are made of diverse traders. The results of our controlled experiments lend support to the Hayek hypothesis [23,25] which asserted that markets can work correctly even though the participants have very limited knowledge of their environment or other participants (see Section 2.1). In fact, traders have different talents, interests and abilities, and they may interpret data differently or be swayed by fads. Nevertheless, there is still room for markets to operate efficiently. These comments on the Hayek hypothesis might also be used to explain why our laboratory markets could be a reasonable approximation of real markets.

Regarding the further development of controlled experiments in the field of experimental econophysics, below I would like to point out the following three directions [130].

- It is necessary to develop controlled experiments to study and predict the movement of real financial/economic markets that are often out-of-equilibrium, [131] which should be helpful for making suggestions for policy makers. For this purpose, it is necessary to take into account the development of complex networks [132–140] that can be used to describe the relations among humans, thus yielding emergent features originating from the interplay between the structure and the corresponding function of markets.
- With the deep development of economic and financial systems in global scope, economic/financial crises have more catastrophic influences than ever. However, mathematical statistics is unable to dig the essential mechanism underlying the crises. This points to a truth: people urgently need a more efficient method to model the social human systems. The controlled experiments, combined with empirical analysis and theoretical analysis, might be a promising candidate. As a result, statistical physics might play a more important role in the prediction of crises.
- Extending controlled experiments from human beings to other animals like ants [141] or fish [142] could be of value in studying statistical physics of other kinds of adaptive agents beyond human beings. By doing so, people could understand those animals more by using concepts or tools originating from the traditional statistical physics.

So far, I have listed only three directions of controlled experiments. The readers are definitely more smart than me, and they might be able to dig out more instructive directions. Certainly, with the development of controlled experiments, accompanying empirical analysis and theoretical analysis should also be developed accordingly, in order to make the experimental results more useful and universal. In the mean time, econophysicists should also try to get a lot of nutrients from other successful disciplines like experimental (or behavioral) economics [26] and social psychology [143], so that experimental econophysics could flourish and prosper as soon as possible.

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