Crash course in Handling networks with MATLAB

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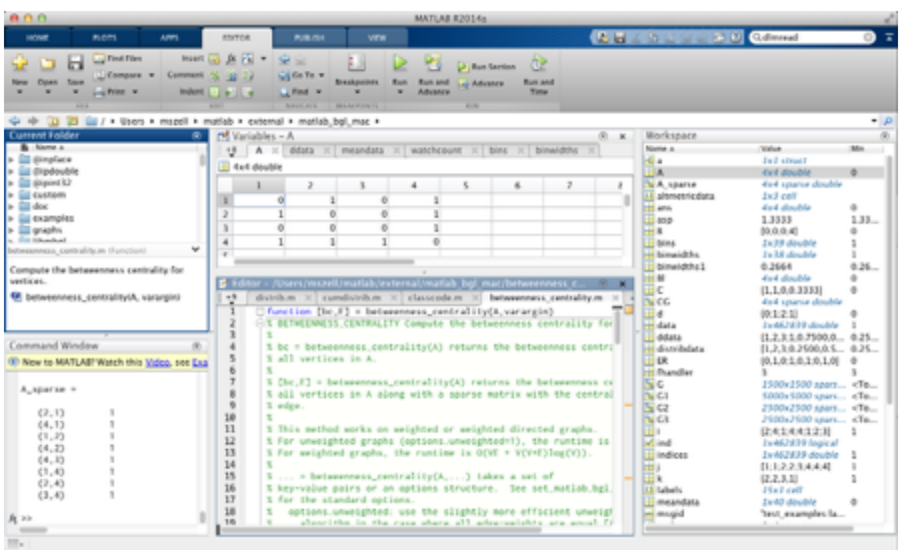
Oct 6th, 2014

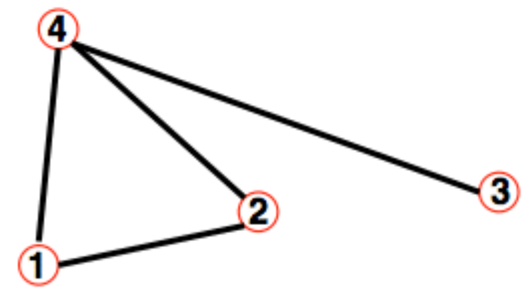
MATLAB is…

… a numerical computing environment and high-level programming language

… fast with matrix manipulation



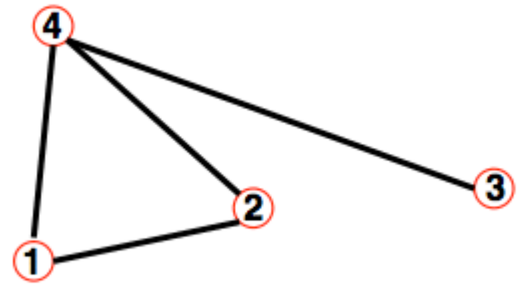
Network representation



Full matrix

```
>> A = [0 1 0 1;1 0 0 1;0 0 0 1;1 1 1 0]
A =
       0      1      0      1
       1      0      0      1
       0      0      0      1
       1      1      1      0
```

Network representation



Sparse matrix

```
>> A = [0 1 0 1;1 0 0 1;0 0 0 1;1 1 1 0];
>> A_sparse = sparse(A)
A_sparse =     (2,1)        1
               (4,1)        1
               (1,2)        1
               (4,2)        1
               (4,3)        1
               (1,4)        1
               (2,4)        1
               (3,4)        1
>> A = full(A_sparse); % Don't do if matrix is large!
```

The “tiny network incident”


```
A_sparse = (1000000,1000001)    1

>> A = full(A_sparse)
?
```

The “tiny network incident”

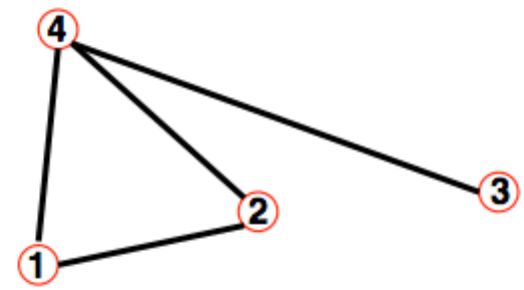

```
A_sparse = (1000000,1000001)     1

>> A = full(A_sparse)
```

(Dramatization)

Make sure your node ids are integers starting at 1

Network representation



Sparse matrix

```
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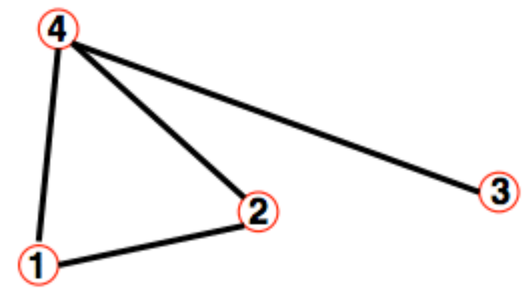
>> A_sparse = sparse(A)
A_sparse =     (2,1)        1
               (4,1)        1
               (1,2)        1
           i   (4,2)  j     1   w
               (4,3)        1
               (1,4)        1
               (2,4)        1
               (3,4)        1
>> A = full(A_sparse); % Don't do if matrix is large!

>> A_sparse = sparse(i,j,ones(size(i)));
>> [i,j,w] = find(A_sparse);
```

Network representation



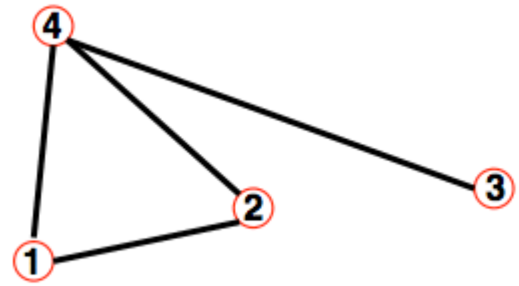
Adjacency list

```
>> node = struct;
>> node(1).neighbors = [2 4];
>> node(2).neighbors = [1 4];
>> node(3).neighbors = [4];
>> node(4).neighbors = [1 2 3];
```

1x4 struct with 1 field

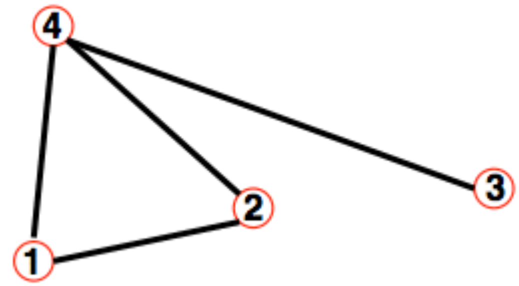
Fields	neighbors
1	[2,4]
2	[1,4]
3	4
4	[1,2,3]

Degree distribution




```
>> A = [0 1 0 1;1 0 0 1;0 0 0 1;1 1 1 0];
>> k = sum(A, 1)
k =   2     2     1     3
```

Degree distribution

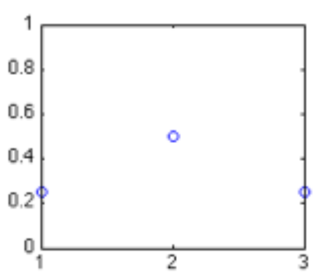



```
>> A = [0 1 0 1;1 0 0 1;0 0 0 1;1 1 1 0];
>> k = sum(A, 1)
k =   2     2     1     3
```

```
function [x,y] = distrib(input)
Your code here
```

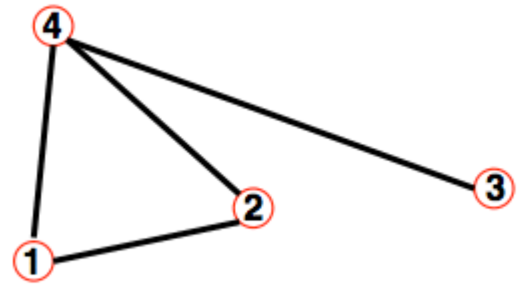
Hint: `unique()`, `histc()`

```
>> [x,y] = distrib(k);
>> plot(x, y, 'o'); ylim([0 1])
```



x:	1	2	3
y:	0.2500	0.5000	0.2500

Cumulative degree distribution



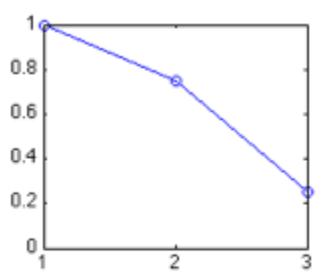

```
>> A = [0 1 0 1;1 0 0 1;0 0 0 1;1 1 1 0];
>> k = sum(A, 1)
k =   2      2      1      3
```

```
function [x,y] = cumdistrib(input)
```

Your code here

Hint: cumsum()

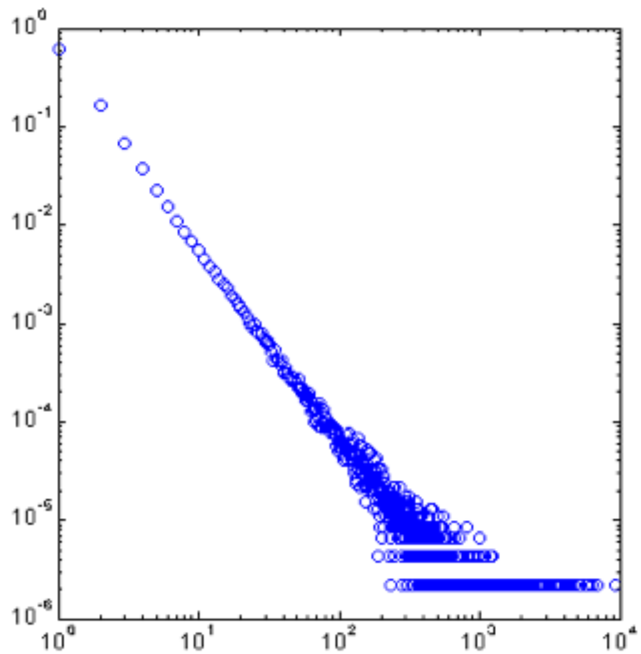
```
>> [x,y] = cumdistrib(k);
>> plot(x, y, '-o'); ylim([0 1])
```



x:	1	2	3
y:	1	0.7500	0.2500

How this looks in a real network

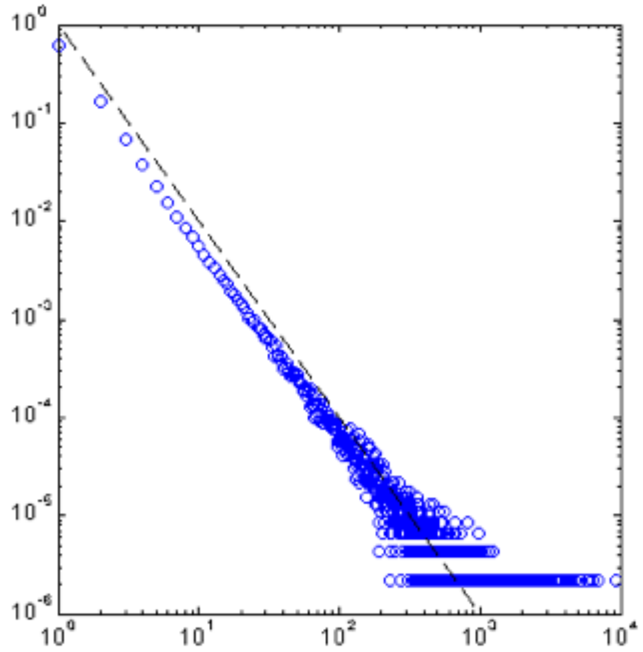
```
>> [x,y] = distrib(k);
>> loglog(x, y, 'o'); hold on
```



How this looks in a real network

```
>> [x,y] = distrib(k);
>> loglog(x, y, 'o'); hold on

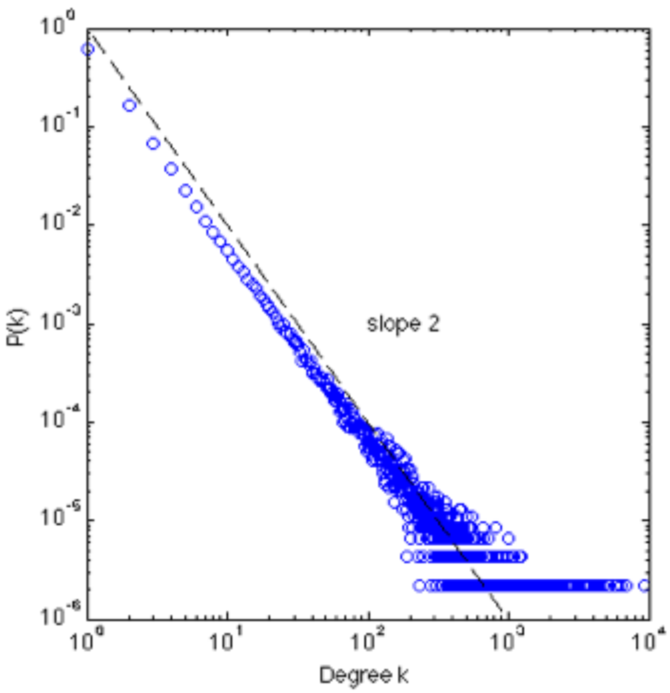
>> loglog([x(1) x(end)], [x(1) x(end)].^(-2), '--k');
```



How this looks in a real network

```
>> [x,y] = distrib(k);
>> loglog(x, y, 'o'); hold on

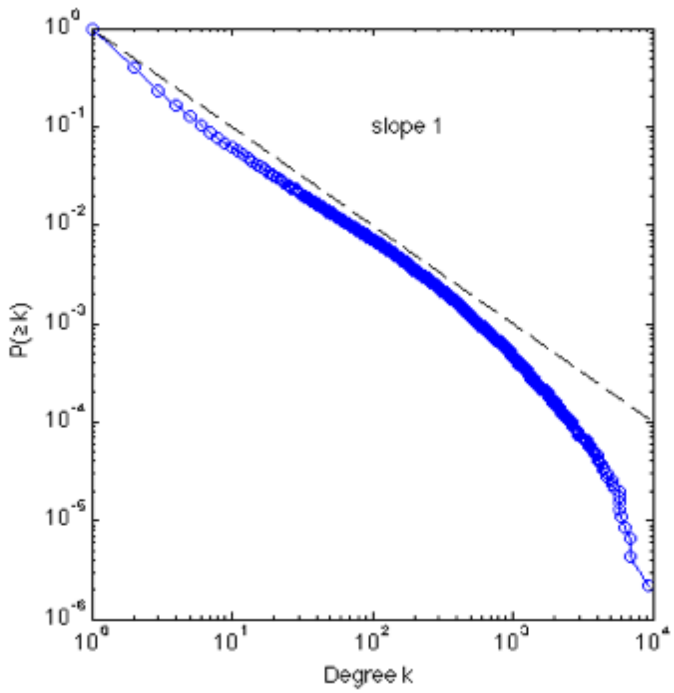
>> loglog([x(1) x(end)], [x(1) x(end)].^(-2), '--k');
>> text(10^2, 10^-3, 'slope 2');
>> xlabel('Degree k'); ylabel('P(k)');
```



How this looks in a real network

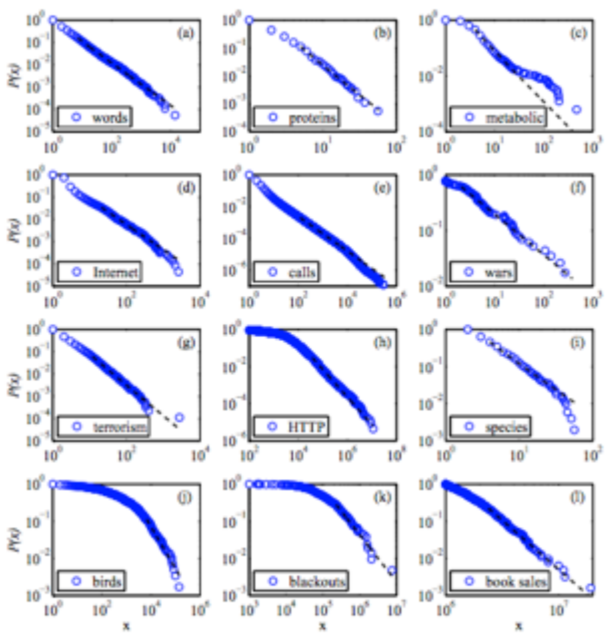
```
>> [x,y] = cumdistrib(k);
>> loglog(x, y, '-o'); hold on

>> loglog([x(1) x(end)], [x(1) x(end)].^(-1), '--k');
>> text(10^2, 10^-1, 'slope 1');
>> xlabel('Degree k'); ylabel('P(\geq k)');
```



Proper power law fitting should use Maximum Likelihood method!

See details & code on Aaron Clauset's website:
http://tuvalu.santafe.edu/~aaronc/powerlaws/



Often "power-laws" are not power-laws

For least squares fitting, use `polyfit()` and `polyval()`

Logbinning (first try)

```
>> bins = logspace(0, 4, 40); % Generate logarithmic bins
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>> pbins = histc(k, bins);    % Count degrees in each bin
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>> bins = logspace(0, 4, 40); % Generate logarithmic bins
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>> pbins = pbins(1:end-1);    % Remove last pbin, due to how histc works
```

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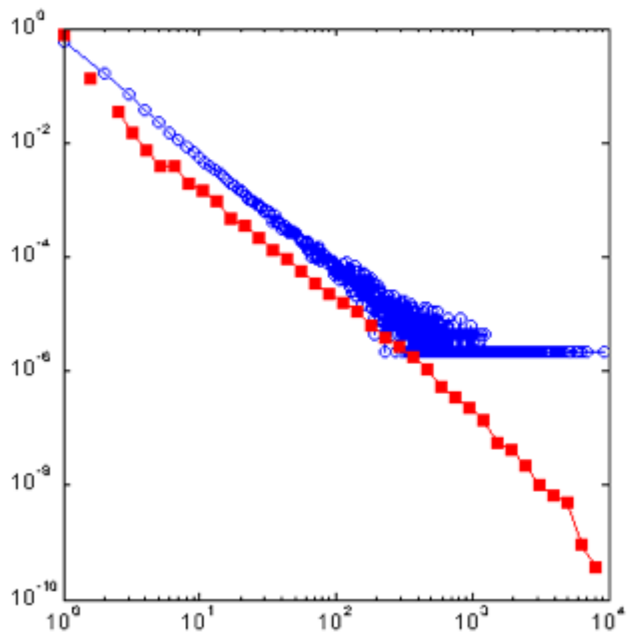
>> bins = bins(1:end-1);      % Remove last bin

>> loglog(x, y, '-o'); hold on % Plot
>> loglog(bins, pbins, '-sr', 'MarkerFaceColor', [1 0 0]);
```



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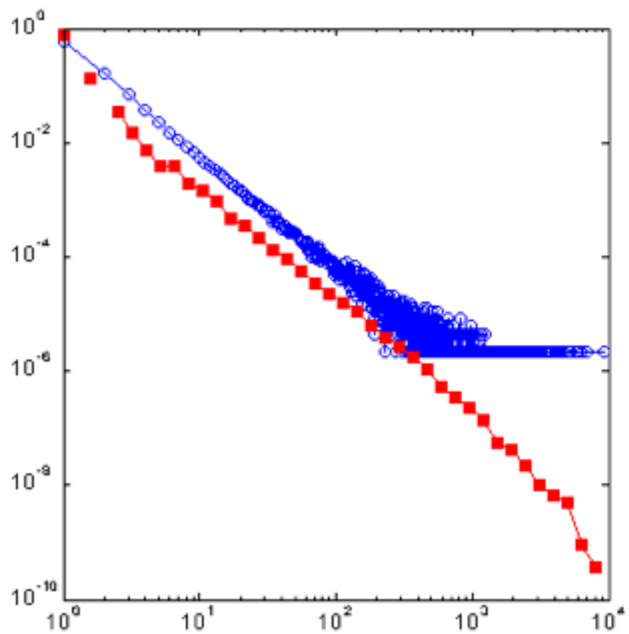


bins:

1	2	3	4	5	6	7	8	9	10
1	1.2664	1.6037	2.0309	2.5719	3.2570	4.1246	5.2233	6.6147	8.3768

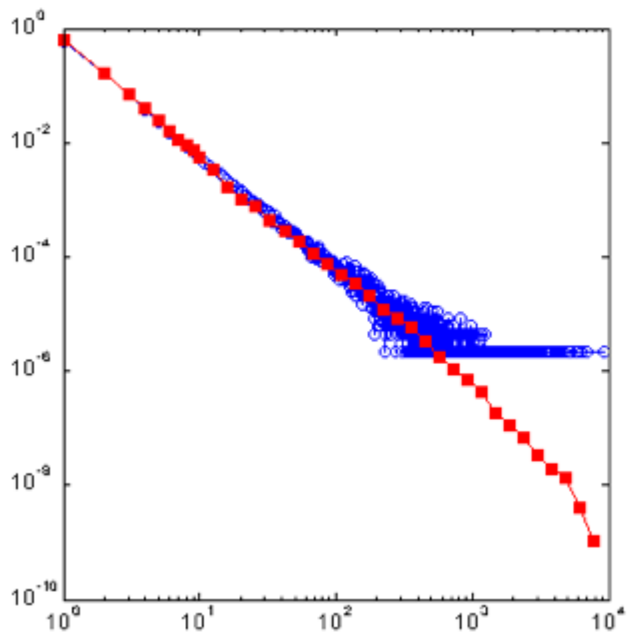
pbins:

1	2	3	4	5	6	7	8	9	10
0.7963	0	0.1327	0	0.0353	0.0150	0.0072	0.0039	0.0039	0.0020

Logbinning

```
>> bins = [linspace(1,9,9) logspace(1, 4, 30)];
>> pbins = histc(k, bins);     % Count degrees in each bin
>> pbins = pbins(1:end-1);     % Remove last pbin, due to how histc works
>> pbins = pbins./diff(bins); % Divide by bin widths
>> pbins = pbins./sum(pbins); % Normalize
>> bins = bins(1:end-1);       % Remove last bin

>> loglog(x, y, '-o'); hold on % Plot
>> loglog(bins, pbins, '-sr', 'MarkerFaceColor', [1 0 0]);
```



bins:

1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10

pbins:

1	2	3	4	5	6	7	8	9	10
0.6370	0.1703	0.0726	0.0390	0.0237	0.0162	0.0115	0.0090	0.0072	0.0055

MatlabBGL: Boost Graph Library is…

…a MATLAB package for working with large graphs

…written in C $\longrightarrow$ very fast!

https://www.cs.purdue.edu/homes/dgleich/packages/matlab_bgl/

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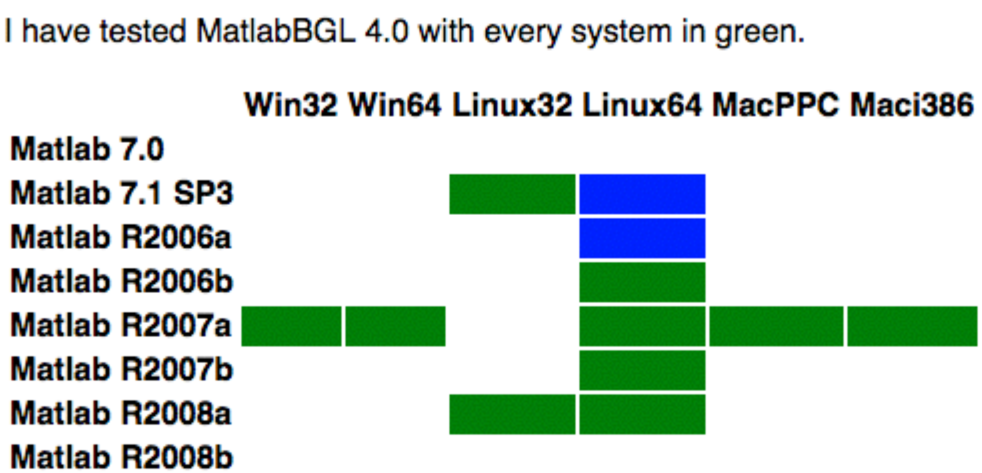
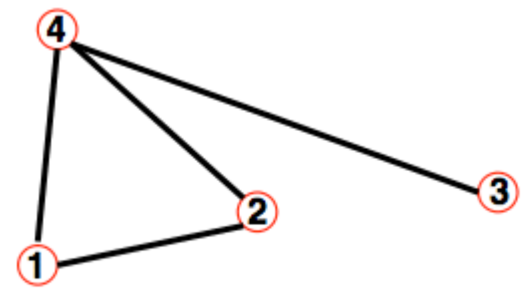


It generally works with other specs

MatlabBGL has most algorithms you will need

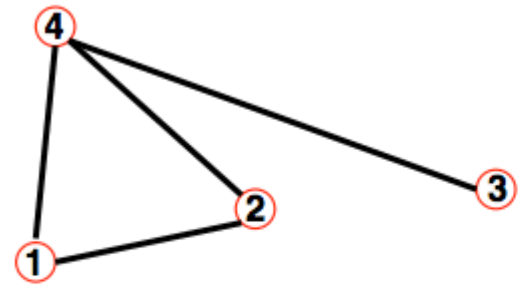
Works with both full and sparse matrices, weighted and directed networks




```
>> C = clustering_coefficients(A)'
C =   1.0000    1.0000         0    0.3333
```

MatlabBGL has most algorithms you will need

Works with both full and sparse matrices,
weighted and directed networks

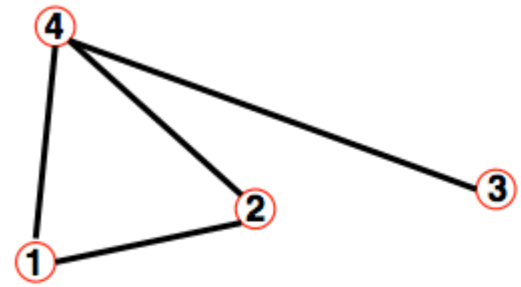



```
>> C = clustering_coefficients(A)'
C =   1.0000    1.0000         0    0.3333

>> SP = all_shortest_paths(A)
SP =
     0     1     2     1
     1     0     2     1
     2     2     0     1
     1     1     1     0

>> asp = mean(SP(SP~=0)) % average shortest path
asp =1.3333
```

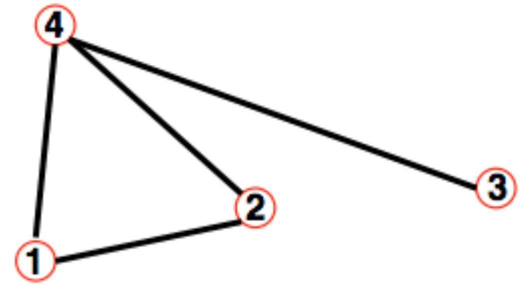
MatlabBGL has most algorithms you will need




```
>> B = betweenness_centrality(A)'
B =   0      0      0      4

>> [B,Bl] = betweenness_centrality(A_sparse); Bl = full(Bl)
Bl =  0      1      0      2
      1      0      0      2
      0      0      0      3
      2      2      3      0
```

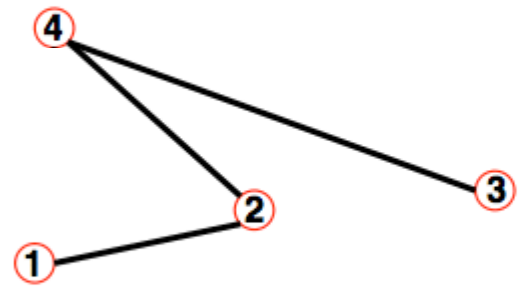
MatlabBGL has most algorithms you will need




```
>> B = betweenness_centrality(A)'
B =   0      0      0      4

>> [B,Bl] = betweenness_centrality(A_sparse); Bl = full(Bl)
Bl =  0      1      0      2
      1      0      0      2
      0      0      0      3
      2      2      3      0

>> T = mst(A) % Minimum spanning tree
T =   (2,1)        1
      (1,2)        1
      (4,2)        1
      (4,3)        1
      (2,4)        1
      (3,4)        1
```



MatlabBGL has most algorithms you will need

```
>> ER = full(erdos_reyni(3,0.5))
ER =  0     1     0
      1     0     1
      0     1     0

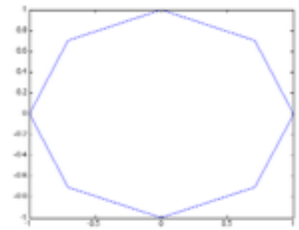
>> SG = full(star_graph(4))
SG =  0     0     0     1
      0     0     0     1
      0     0     0     1
      1     1     1     0

>> CG = cycle_graph(4)
CG =  (2,1)        1
      (3,2)        1
      (4,3)        1
      (1,4)        1
```

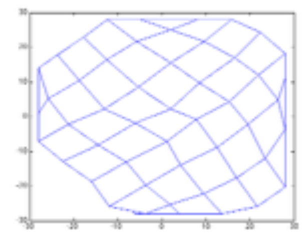
and much more…

MatlabBGL also implements simple layouts

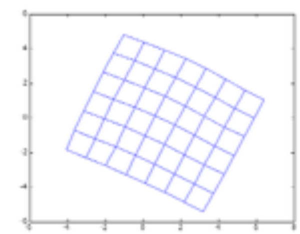
```
% circle_graph_layout
G = cycle_graph(8);
X = circle_graph_layout(G);
gplot(G,X);
```



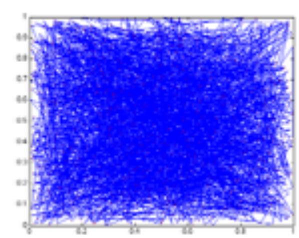
```
% fruchterman_reingold_force_directed_layout
G = grid_graph(8,7);
X = fruchterman_reingold_force_directed_layout(G);
gplot(G,X);
```



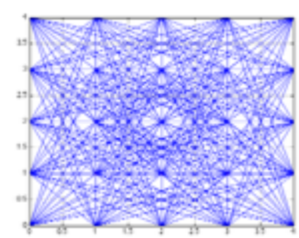
```
% kamada_kawai_spring_layout
G = grid_graph(8,7);
X = kamada_kawai_spring_layout(G);
gplot(G,X);
```



```
% random_graph_layout
G = cycle_graph(1500);
X = random_graph_layout(G);
gplot(G,X); hold on; plot(X(:,1),X(:,2),'r.'); hold off;
```



```
% Layout on the grid
X = random_graph_layout(G,int32([0 0 5 5])); % random grid layout
gplot(G,X); grid on; hold on; plot(X(:,1),X(:,2),'r.'); hold off;
```



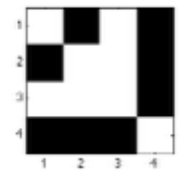
More useful MATLAB functions for working with data/networks

Manual

http://www.mathworks.com/help/matlab/

`triu()`, `tril()`: Upper/Lower triangular matrix

`imagesc()`: Scale data and display image



`eig()`: Eigenvalues and eigenvectors

`accumarray()`: Like an SQL "GROUP BY"

`load()`, `save()`, `fscanf()`, `fprintf()`, `csvread()`, `csvwrite()`, `dlmread()`, `dlmwrite()`: Read and write data from/to files

Example: Loading the lesmiserables data

```
>> el = dlmread('edge_list.csv', '\t', 1, 0); % read data (skip header)
>> el = [el; el(:, [2 1 3])]; % make undirected
>> A = sparse(el(:, 1)+1, el(:, 2)+1, el(:, 3)); % +1 to ids to start at 1
```