

A Simple Dynamical Model of the Stock Market

Rashi Verma and Rajita Menon

Department of Physics, Boston University, Boston, MA 02215, USA

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This paper presents an overview of a study of connections between stocks in a simple economic model, under different conditions and changing parameters. The fluctuating psychological factors influencing stock prices are taken into account in a novel way, by assigning certain rules for distributing stock prices.

Introduction

It is difficult to generate a faithful model of the stock market that is easy to simulate, due to the dependence of stock prices on a huge number of variable factors. There may be several approaches to the problem, and there have been studies that treat investors as percolation clusters to find the statistics of price variations [2]. In general, models of the stock market tend to be highly complex, and there is a need to establish a model that captures the essential behavior in a minimalistic form, while reducing the explicit inclusion of the microscopic details that make these systems so chaotic. Constructing such a model in order to examine the dynamics of a small system of stocks is the primary aim of the present discussion. The behavior of the system can be analyzed to study the effects of inter-stock correlations, by establishing an underlying directional network of pushes and pulls between the stocks, which influences their interactions in the model. The ultimate goal is to carry out an exhaustive analysis of real world data from the stock market to test the efficacy of this skeletal model to predict market instabilities.

Motivation for the Model

The details of the model and simulation will be introduced in the next section, but the idea that the market follows some sort of self organization is pertinent to the discussion. It seems worthwhile to explore the connection between self organized criticality as observed in sandpile models and the OFC model for earthquakes [1] and stock market cascades. One can think of the system as being composed of a number of nodes, or clusters of nodes, that each represent a

stock. Within this system of stocks, there exist external and internal stimuli that tend to push prices of individual stocks up or down based on the response of investors to these stimuli. Modeling the changes in prices while accounting for the effects of these stimuli can be done by introducing a blanket parameter known as the *degree of undesirability* (DOU) of a particular stock. This parameter essentially includes all sorts of external stimuli that may negatively impact the demand for a certain stock in the market. There are a number of events such as natural disasters, political instability, economic crises, social movements, or any other conceivable eventuality, local or global, that can change the trading inclinations of buyers and sellers. The implicit assumption is that these stimuli are inextricably tied up with the behavior of traders, and their effects may be re-interpreted in terms of psychological factors underpinning the rise and fall of stock prices.

Relevance of the DOU

Since the 1960s, Eugene Fama's Efficient Market Hypothesis (EMH) has been influential in studies of the stock market. It asserts that financial markets are 'informationally efficient'. In consequence of this, one cannot consistently achieve returns in excess of average market returns on a risk-adjusted basis, given the information available at the time the investment is made. The EMH assumes that psychological behaviors 'cancel out' overall, which might bring into question the requirement of a parameter like the DOU which accounts for psychological biases. However, there has been significant evidence that these biases may result in statistically anomalous stock price movements. For

instance, stocks with low price to earnings (and similarly, low price to cash-flow or book value) outperform other stocks [5] [4]. Alternative theories have proposed that cognitive biases cause these inefficiencies, leading investors to purchase overpriced growth stocks rather than value stocks [3]. Psychological research has demonstrated that people are predisposed to seeing patterns, and often will perceive a pattern in what is, in fact, just noise. In the present context this means that a succession of good news items about a company may lead investors to overreact positively, which would correspond to a decrease in the stock DOU (unjustifiably driving the price up).

In normal times the market behaves like a game of roulette; the probabilities are known and largely independent of the investment decisions of the different players. In times of market stress, however, the game becomes more like poker, with herding behavior and group dynamics taking over. The players now must give heavy weight to the psychology of other investors and how they are likely to react psychologically. It seems, therefore, that an inclusion of the effects of psychological factors in the form of the DOUs is justified.

Bi-Directional Network of Stocks

Having characterized each stock using the DOUs, the next task is to establish the meaning of the directional network connecting the stocks/nodes. The nodes are connected to each other with links of variable strength, which signify inter-connectedness based on their specific functionalities. This network dictates how changes in the DOU of one of the stocks affects others connected with it, depending on the weight and the sign of the connecting link. Therefore, we are able to account for the fact that two stocks can be correlated positively (with an increase(decrease) in the DOU for one stock causing a corresponding increase(decrease) in the DOU for the second stock) or negatively (with an increase(decrease) in the DOU for one stock causing a corresponding decrease(increase) in the DOU for the second stock).

Model Setup

Our model is roughly inspired from the Olami-Feder-Christensen (OFC) model for earthquakes [1]. We begin with a 1D lattice of n stocks, each stock i having a certain DOU value D_i assigned to it (analogous to stresses on earthquake sites). Simultaneously, there is a small residual value of DOUs D_i^{res} assigned to every stock, while the threshold value D_i^{max} of each stock is set to 1. There is also another parameter which governs stress/DOU transfer, the directional network C set up connecting each stock i to every other stock j with a C_{ij} value between -1 and 1 .

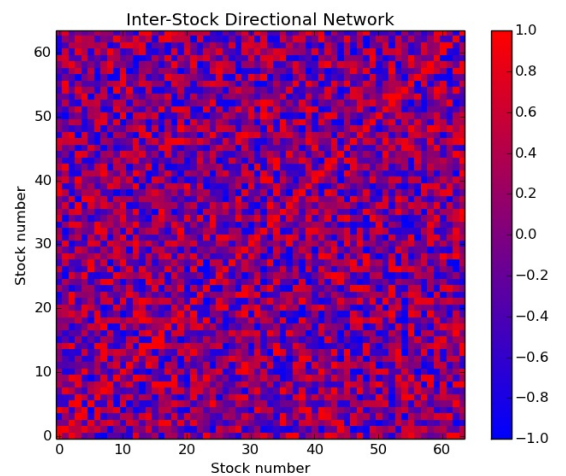


FIG. 1: Stock network heat map

The idea is that any stock that goes beyond this value can be thought of as having crashed (or equivalently, being supercritical), and must relieve its DOUs to the other stocks. This can be thought of as a fully connected long range model, since each supercritical stock relieves its DOUs to every other stock, based on the directional network (C); e.g., if a stock i becomes supercritical, it will transfer some fraction (α) of its DOU to the j th stock in the following way:

$$D_j = D_j + C_{ij}\alpha(D_i - D_i^{res}); \quad i \neq j = 1 \dots n \quad (1)$$

and then be set to its residual value $D_i = D_i^{res}$. α is usually fixed at the beginning of the simulation, and physically it represents the DOU

losses to the system occurring during transfers. Having assigned the initial D and C values, the first market cycle ($m = 1$) can begin. A small random noise is added in the beginning to each D_i . This might represent general temporal fluctuations. The stock with the maximum DOU is then found, and if subcritical even after adding noise, is taken to criticality (i.e., set equal to 1) (referring to an external undesirable event that strongly affected a single susceptible stock). Its excess DOU value is then dissipated to every other stock, based on equation (1). This DOU release to the other stocks might in turn cause some more stocks to fail, which would then release their excess DOUs to all stocks in the same way. So, this cascade effect continues until every stock becomes subcritical again. At this point, a single market cycle has concluded, and the next one can begin with the addition of noise. Each market cycle represents the rearrangement of a supercritical system back to a pseudo-equilibrium state, where the stocks are relatively stable. The simulation is continued for m market cycles.

Note that since some C_{ij} values are negative too, the D_i s might go lower than the residual values. In that case, those DOUs are set equal to D_i^{res} s. In this way, the DOUs are always maintained between 0.25 and 1 at the end of each market cycle.

Simulation Details

Fixed parameters:

$$\alpha = 0.9$$

$$D_i^{max} = 1; \quad D_i^{res} = 0.25 \pm 0.05$$

$$C_{ij} = (-1, -1]$$

Changing parameters:

$$D_i = [D_i^{res}, D_i^{max}]$$

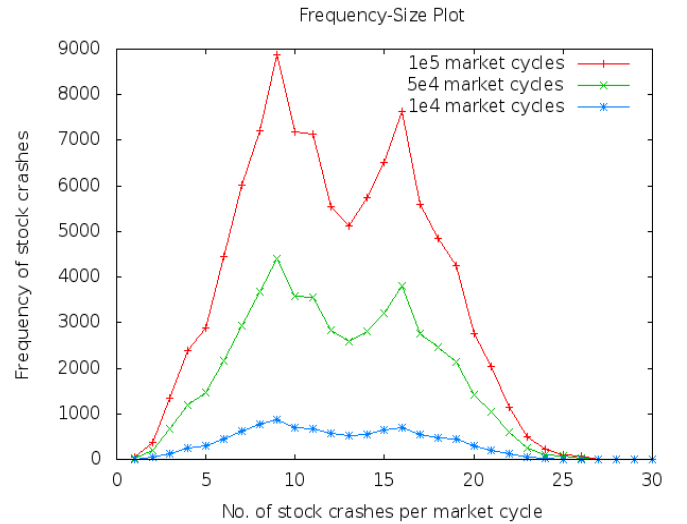
(drawn from a uniform distribution at step 0)

$$D_i \geq D_i^{res} \text{ in the subsequent steps}$$

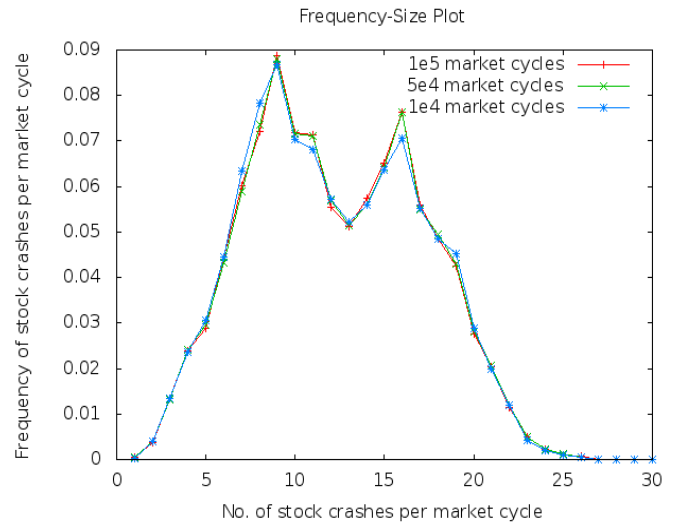
Number of Market Cycles (m)

$$= 10^4, 5 \times 10^4, 10^5, 10^6$$

$$\text{Number of Stocks } (n) = 64, 100$$



(a)



(b)

FIG. 2: System of 64 stocks

Analysis

We ran our simulations on market systems of various sizes (32, 64, 100, 200), and the graphs obtained for market sizes 64 and 100 will be discussed in this section. It seems to be of interest to plot a frequency size distribution, as well as a rank size distribution for these markets, in order to explore universality and scaling properties and investigate the effect of inter-stock correlations on rank size distribution. We will also try to examine the dynamics of positively and negatively correlated stocks through varying numbers of market cycles.

Frequency-Size Plots

The first point of interest is to extract some kind of a *Zipf law* from the problem. To this end, the number of stock crashes per market cycle were counted, irrespective of the stock label (each stock can fail multiple times during the course of a single market cycle). Therefore, for a certain number of market cycles, information about the number of stock crashes per market cycle was recorded and plotted versus the frequency of occurrence of that particular crash event for market size 64 and 100.

Figure 2(a) plots the frequency size distribution with simulations carried out for 10^4 , 5×10^4 , 10^5 market cycles. The three curves have been rescaled (division by number of market cycles m) in 2(b), and collapse to a single curve indicating universality. Figure 3(a,b) plots the frequency size distribution on log-log axes. We observe a *bimodality* in these plots, with the frequencies peaking at two distinct crash events.

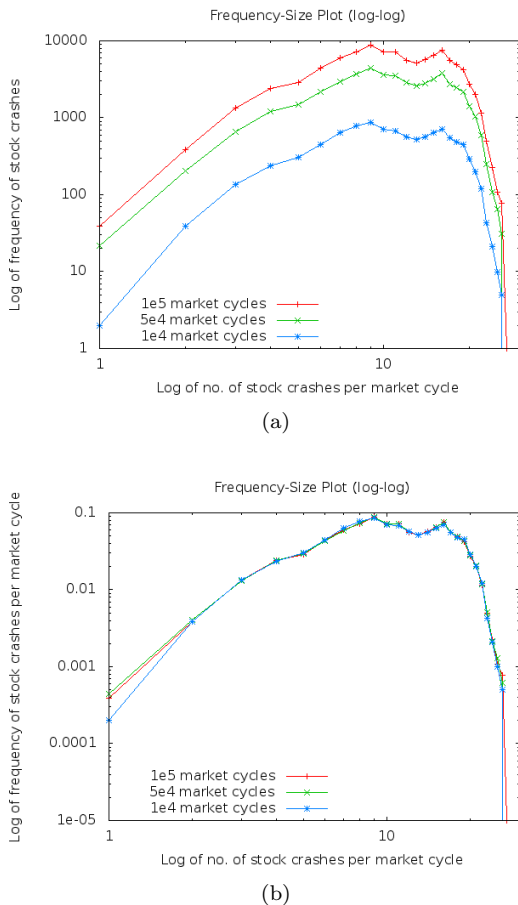


FIG. 3: System of 64 stocks

The frequency size distribution for 100 stocks is shown in Figures (4) and (5). These plots exhibit behavior that is qualitatively similar to the 64 stock system. However, we note that the bimodality is less pronounced for this larger market. This indicates the possibility of a change in behavior of the market as we cross over from the small market regime to the large market regime.

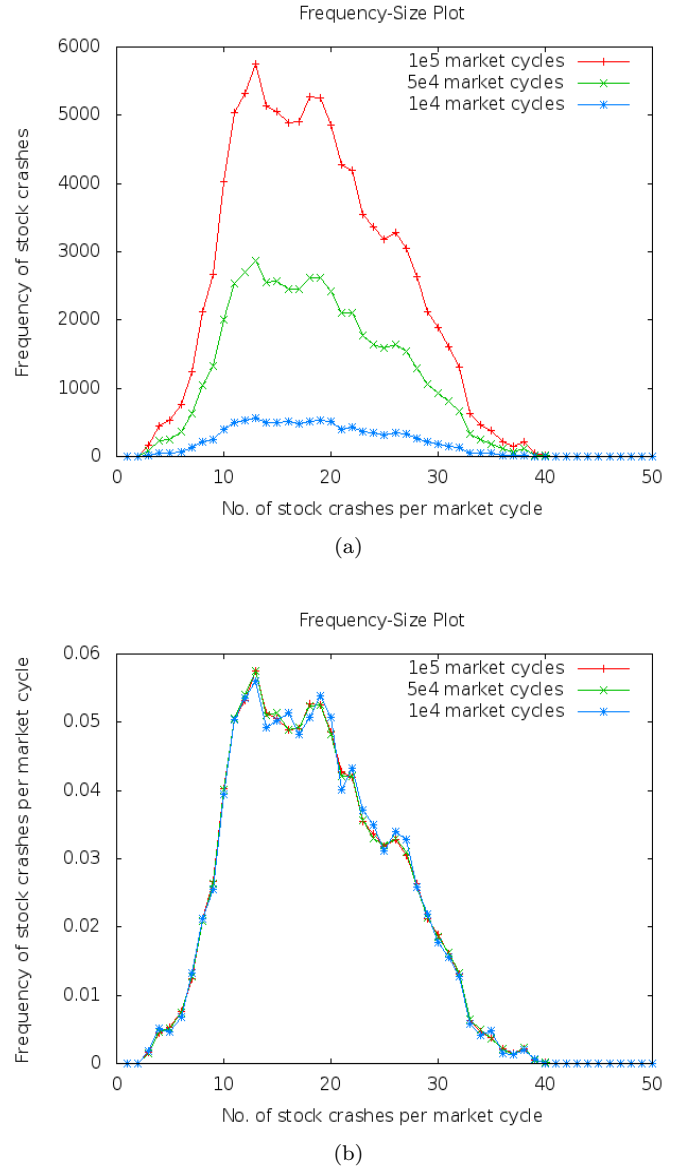


FIG. 4: System of 100 stocks

The bimodality of the frequency size distribution also seems to be dependent on the number of market cycles m . As m increases, a plateau effect is observed, with a large number of different crash events becoming more and more likely.

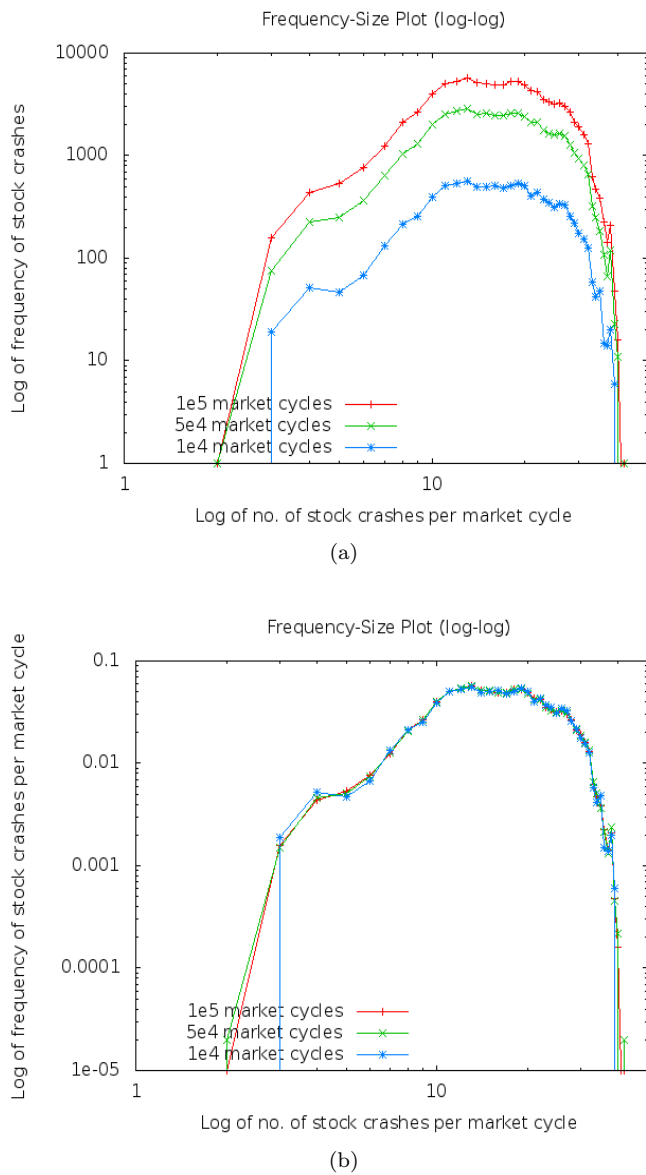


FIG. 5: System of 100 stocks

Rank Size Plots

The rank size distribution for market sizes 64 and 100 are shown in Figures (6) and (7). We do not observe a power law behavior for the entire distribution. The curve transitions from a slowly changing region to a regime of sharp decrease. The tail of the distribution appears to exhibit a power law, with a similar slope for each of the curves.

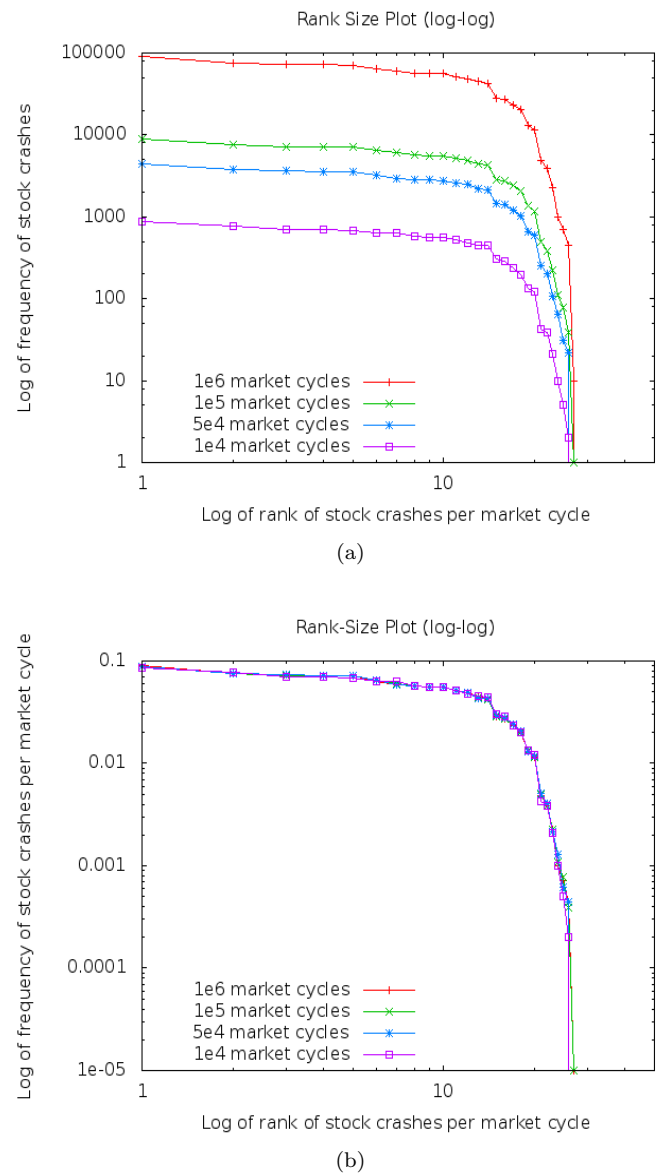


FIG. 6: System of 64 stocks

Discussion of bimodality

The frequency size distribution for markets comprising 32, 64, 100 and 200 stocks have been plotted in Figure (8). The bimodality of crash events in these distributions becomes progressively weaker as the market size increases, and this weakening is monotonic from the 32 stock system to the 200 stock system.

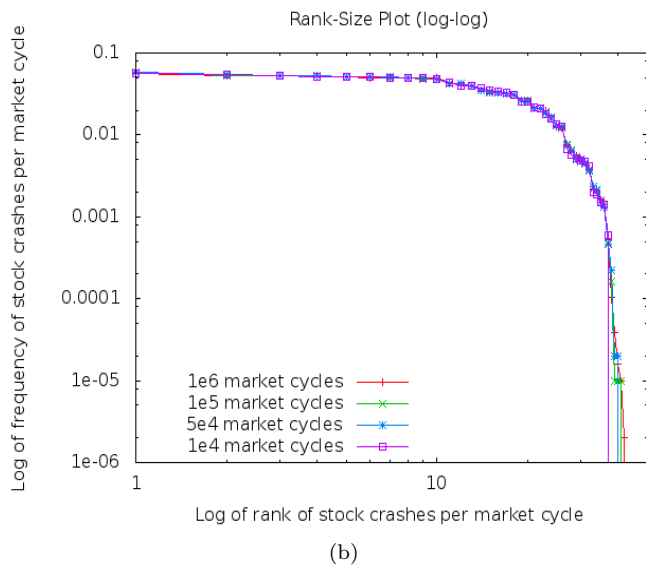
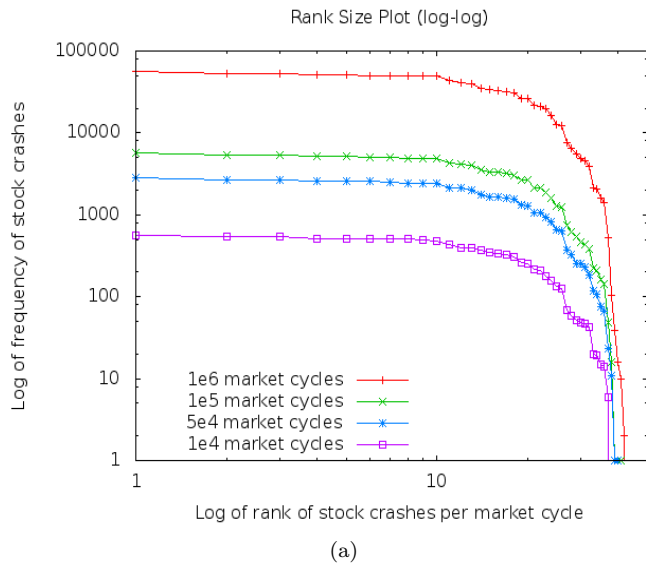


FIG. 7: System of 100 stocks

This indicates that larger systems do not exhibit a preference for a few crash events, once again a plateau is seen with equal likelihood for many different crash events. On the other hand, small systems prefer certain distinct crash events, which have been labeled in the figure. It would be interesting to determine the market size cutoff beyond which the bimodality completely disappears. In some sense, this can be interpreted as a phase transition for the crash events.

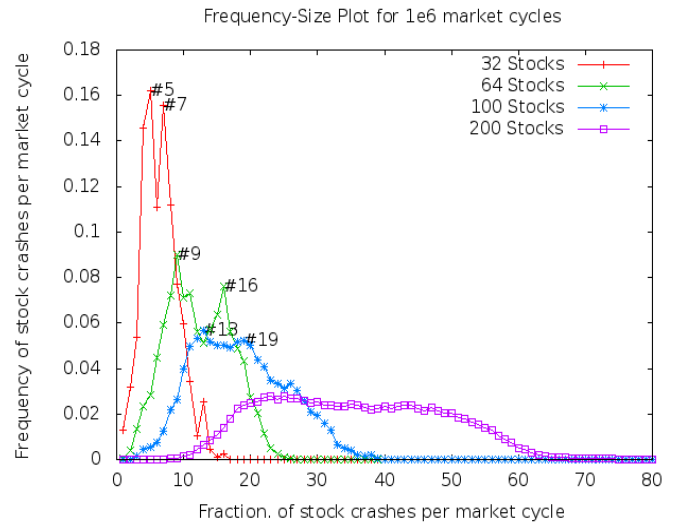


FIG. 8: Bimodality

Scope for Further Study

We have considered a fully connected directional network between stocks, which makes it difficult to quantify the effects of a positive or negative correlation between 2 stocks on the evolution of the DOUs of either stock. Figure (9) shows the time evolution of DOUs for pairs of strongly connected stocks, and was plotted to detect patterns of pushes or pulls based on whether the connection is positive or negative. However, the complexity of the network makes it impossible to isolate the effects of these correlations, and the graphs seem very chaotic. A sparse directional network needs to be considered, for such an analysis to yield sensible results.

The following are some modifications to our model that may be considered for further study

1. Consider a sparse correlation matrix to isolate the effects of single stock correlations.
2. Examine the effects of generating the correlation matrix from different distributions on the rank size graph.
3. Quantify the relationship between correlations and stock crashes by establishing a suitable parameter.
4. Generate labels for stocks by drawing the corresponding *DOUs* from different distributions.

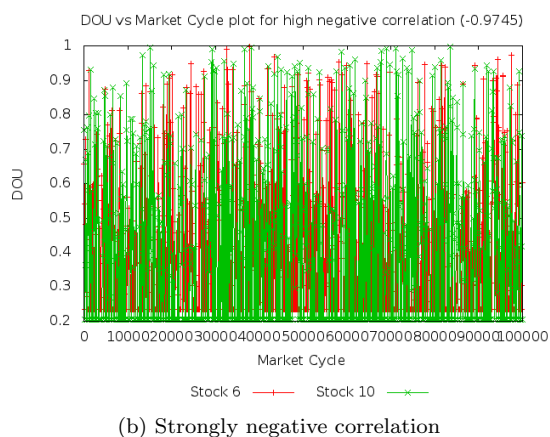
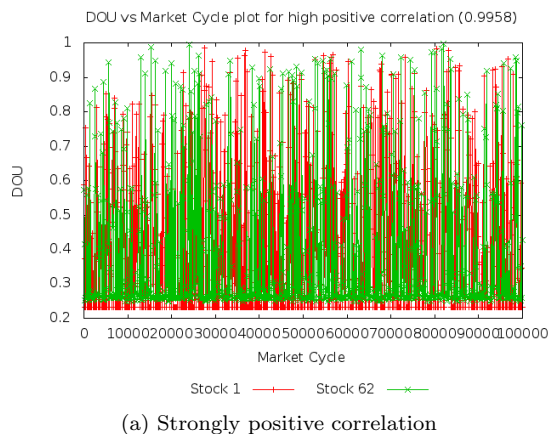


FIG. 9: Time evolution of DOUs for stock pairs

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