A Quantum-Mechanics Framework of Dynamic Economics

Xiangyi Meng

Jing Ma

(EECS & School of Physics, Peking University)

Department of Physics, Boston University

Motivation & Outline

Motivation

Dynamic Economics [1]

Application of Quantum Mechanics in Finance [2]

Business Cycles

Time Series

Probability Theory

Option Pricing

Dynamic Systems

Path Integral and Futures

Quantum Field Theory and Interest Rates

Hamiltonian Systems

[1] T. J. Sargent, *Dynamic Macroeconomic Theory* (Harvard University Press, 1987).

[2] B. E. Baaquie, *Quantum Finance: Path Integrals and Hamiltonians for Options and Interest Rates* (Cambridge University Press, 2004).

Motivation & Outline

Outline

Motivation & Outline

Framework

Summary

- **Dynamic Economics**
  1. Equilibrium Price
  2. Damped Harmonic Oscillator
- **Irrationality**
  1. Quantum Harmonic Oscillator Model
  2. Quantum Brownian Motion Model (qBm(m))
- **Data Analysis**
  1. "Fat Tail" and Non-Markovianity
  2. Kurtosis and Autocorrelation of qBm(m)
  3. Additional Less-Colored Noise

Dynamic Economics

Equilibrium Price

The equilibrium price is determined by the demand and supply [1], but how? (what about the timescales?)

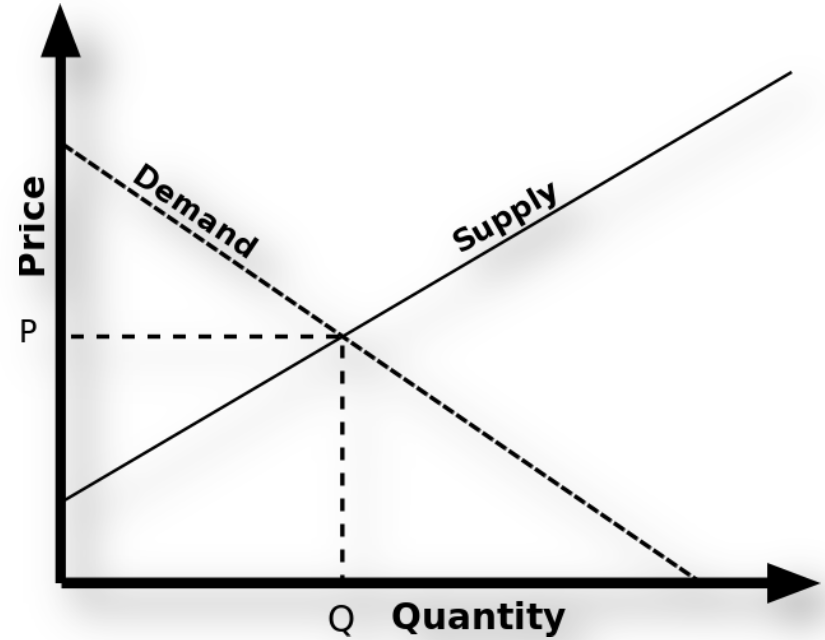
Dynamic problems in economics can be sorted into two classes: propagation and impulsion [2].

1) How does external information propagate in a given financial structure?

(theory of dynamic systems)

2) What is the statistical behavior of impulsion?

(theory of stochastic process)



[1] T. J. Sargent, *Dynamic Macroeconomic Theory* (Harvard University Press, 1987).
[2] R. Frisch, in *Economic Essays in Honor of Gustav Cassel* (George Allen & Unwin, London, 1933), pp. 171.

Dynamic Economics

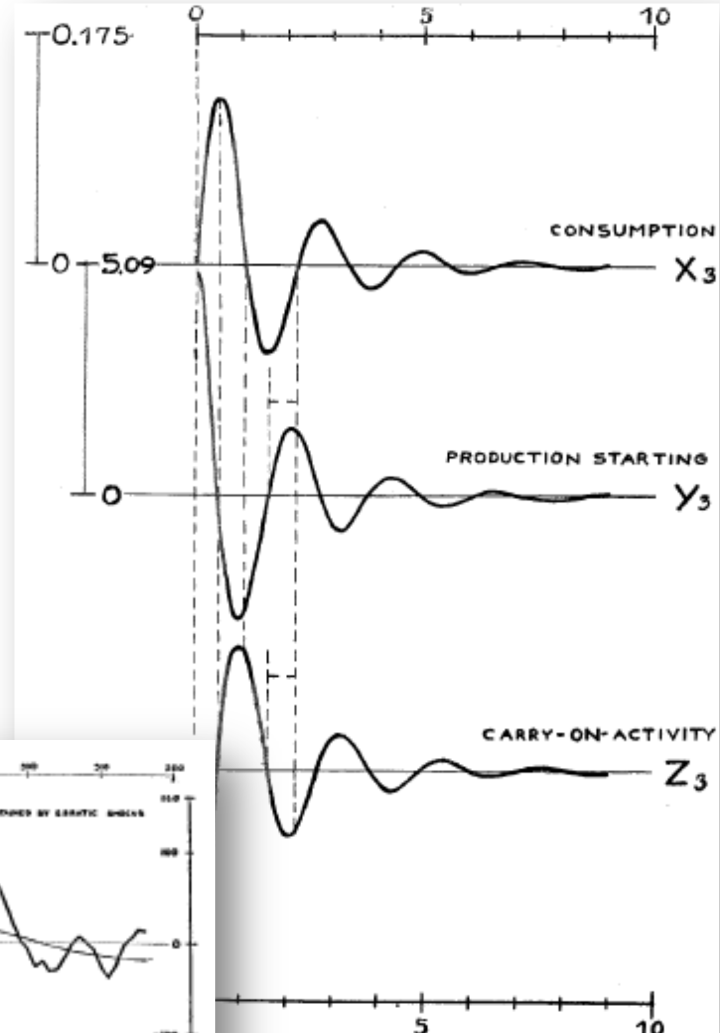
Damped Harmonic Oscillator

Damped Harmonic Oscillator [1]

The propagation problem can be modeled by a **damped harmonic oscillator** (overdamped or underdamped). New equilibriums can be reached within the mechanism

External (Noise) Reservoir [1]

External **stochastic** impulsion produces random patterns, which are similar to **Brownian noise**.



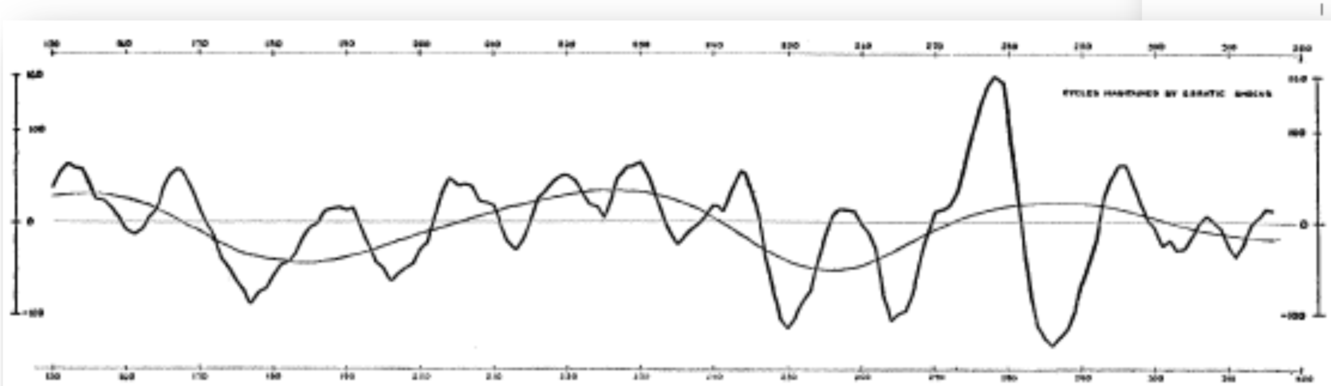


Fig. 6

[1] R. Frisch, in *Economic Essays in Honor of Gustav Cassel* (George Allen & Unwin, London, 1933), pp. 171.

Irrationality

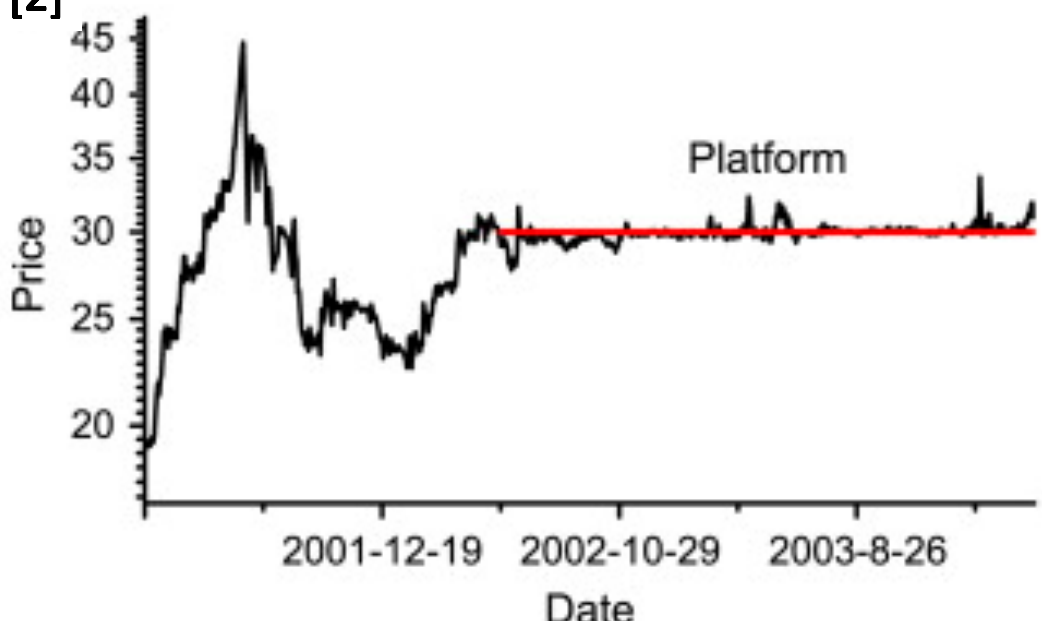
Quantum Harmonic Oscillator Model

Persistent Fluctuation

The classical model cannot explain why there exists **persistent fluctuation** of price [1,2].

The external (noise) reservoir would be **too large** if the persistent fluctuation was all produced by stochastic information [1].

[2]



[1] P. Chen, in *Nonlinear Dynamics and Economics*, edited by W. A. Barnett, A. P. Kirman, and M. Salmon (Cambridge University Press, Cambridge, 1996), pp. 307.

[2] C. Ye and J. P. Huang, Physica A **387**, 1255 (2008).

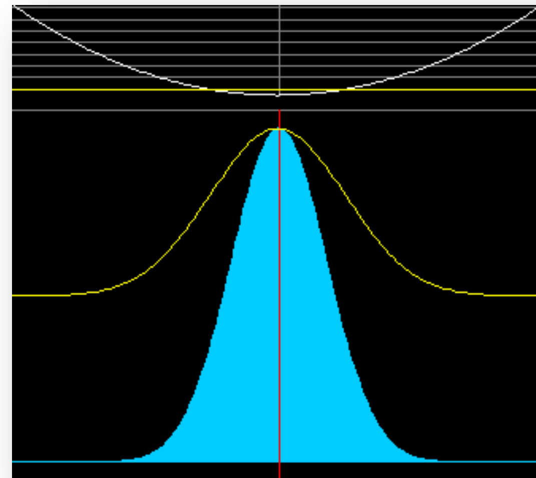
Irrationality

Quantum Harmonic Oscillator Model

Quantum Harmonic Oscillator Model [1]

A microscopic explanation of **persistent fluctuation:**

If transactions are completely rational, the price should be determined with **certainty**; however, the irrationality of transaction will introduce additional fluctuations of the price and thus will lead to finite small but persistent **uncertainty** [2].



Ground State
(Gaussian wave packet)

$$|\varphi_0(x)|^2 = \sqrt{m\omega/\pi\hbar}\,\exp(-m\omega x^2/\hbar);$$

$$\sigma_x^2 = \hbar/2m\omega,$$

$$E = \hbar\omega/2.$$

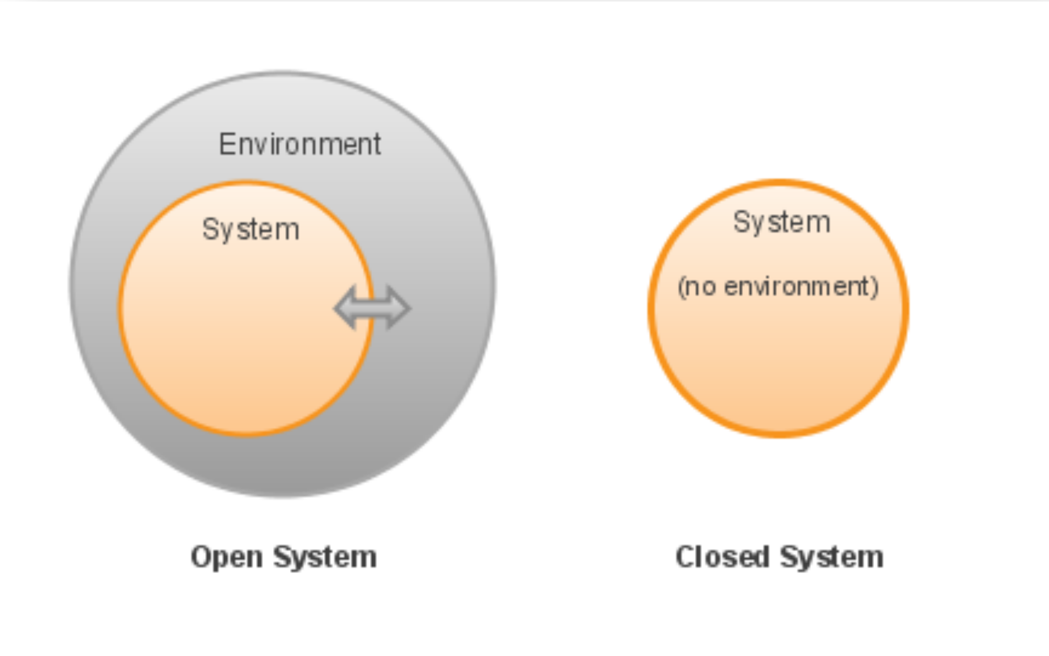
Quantum Closed System	Single Stock
Coordinate representation X_i	(logarithmic) Stock price $\ln s_i$
Momentum representation P_i	Trend of stock price $m_i\, d(\ln s_i)/dt$
Mass m_i	Inertia of stock i
Energy E_i	Trading volume of stock i
Wave function (amplitude) $\lvert\varphi_i(x,t)\rvert^2$	Probability density distribution of price
Uncertainty relation $[X,P]=i\hbar$	Uncertainty of irrational transaction

[1] C. Ye and J. P. Huang, Ph
[2] X. Meng, J.-W. Zhang, H.

Irrationality

Quantum Brownian Motion Model (qBm(m))

Quantum Open System [1]



In physics, an **open quantum system** is a quantum-mechanical system which interacts with an external quantum system (with a large number of degrees of freedom), the **environment.**

In reality, every quantum system is open to some extent, causing **dissipation and fluctuation** in the quantum system.

For example: **spontaneous emission** in quantum optics (dissipation of photons).

[1] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, 2002).

Irrationality

Quantum Brownian Motion Model (qBm(m))

Quantum Brownian Motion Model

We regard the stock index as a free particle interacting with a large number of single stocks–a **thermal reservoir** [2].

The Hamiltonian of qBm:

$$H = H_A + H_E + H_I = \frac{1}{2M} P^2 + V(X) + \sum_i \left(\frac{1}{2m_i} p_i^2 + \frac{1}{2} m_i \omega_i^2 x_i^2 \right) - X \sum_i \kappa_i x_i .$$

The Caldeira-Leggett master equation [1]:

$$\frac{d}{dt} \rho_A(t) = -\frac{i}{\hbar} [H_A, \rho_A(t)] - \frac{i\gamma}{\hbar} [X, \{P, \rho_A(t)\}] - \frac{2M\gamma kT}{\hbar^2} [X, [X, \rho_A(t)]].$$

Mapping between quantum open system and stock index [2]

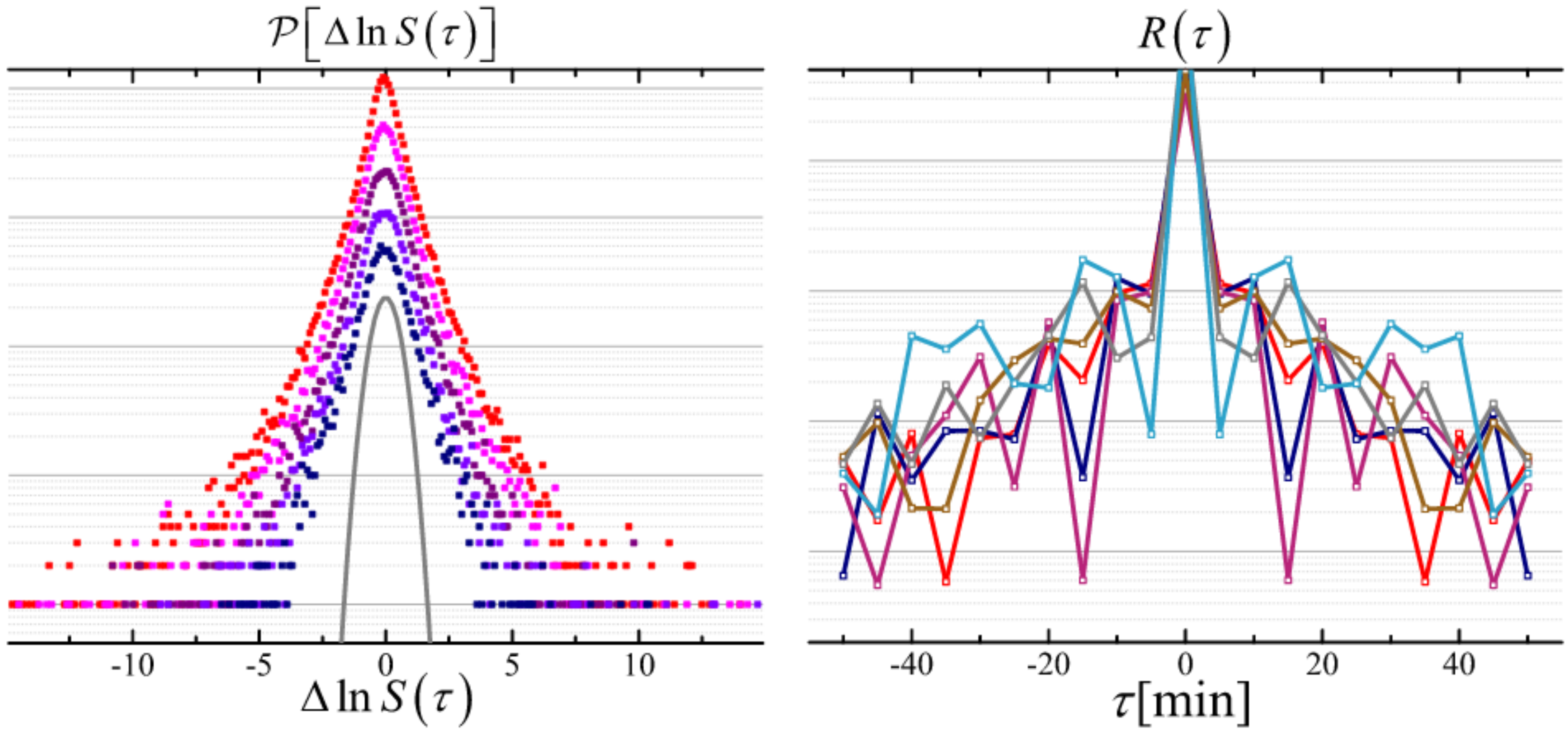
Quantum Open System	Stock Index
Density operator (population) $\rho_A(x,x,t)$	Probability density distribution of stock index
Potential well $V(x)$	Macroscopic external influence on stock index
Thermal reservoir ρ_E	Large numbers of stocks
Temperature kT	Strength of fluctuation
Dissipation coefficient γ of thermal reservoir	Strength of dissipation
Spectral density $J(\omega)$ of thermal reservoir	Autocorrelation features

[1] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, 2002).

[2] X. Meng, J.-W. Zhang, H. Guo, Physica A **452**, 281 (2016).

Data Analysis

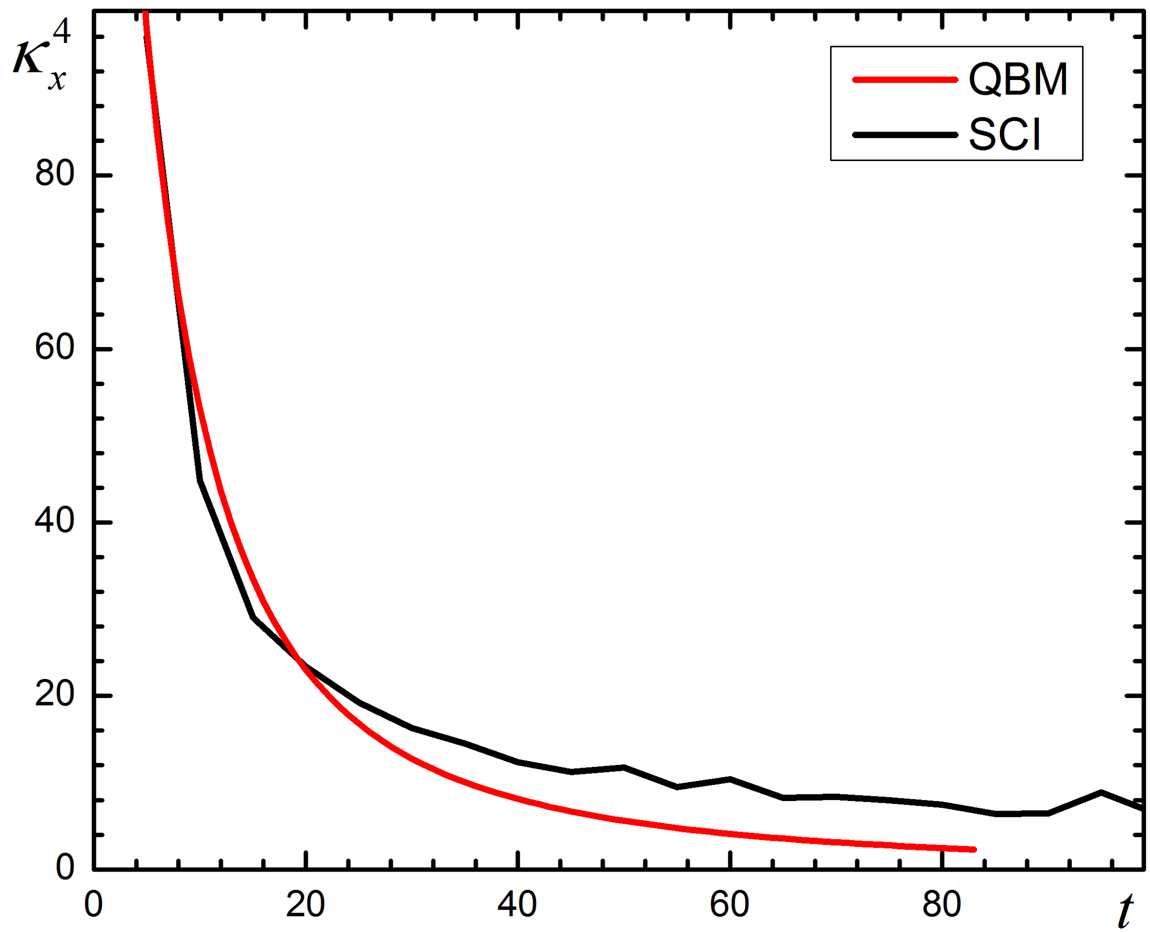
"Fat Tail" and Non-Markovianity



a) Non-Gaussian distribution of log return $\Delta \ln S(\tau)$ -> **the kurtosis is larger than 0.**
b) Non-zero autocorrelation (non-Markovianity) $R(\tau)$, not a Brownian motion
-> **business cycles exist.**

Data Analysis

Kurtosis and Autocorrelation of qBm(m)



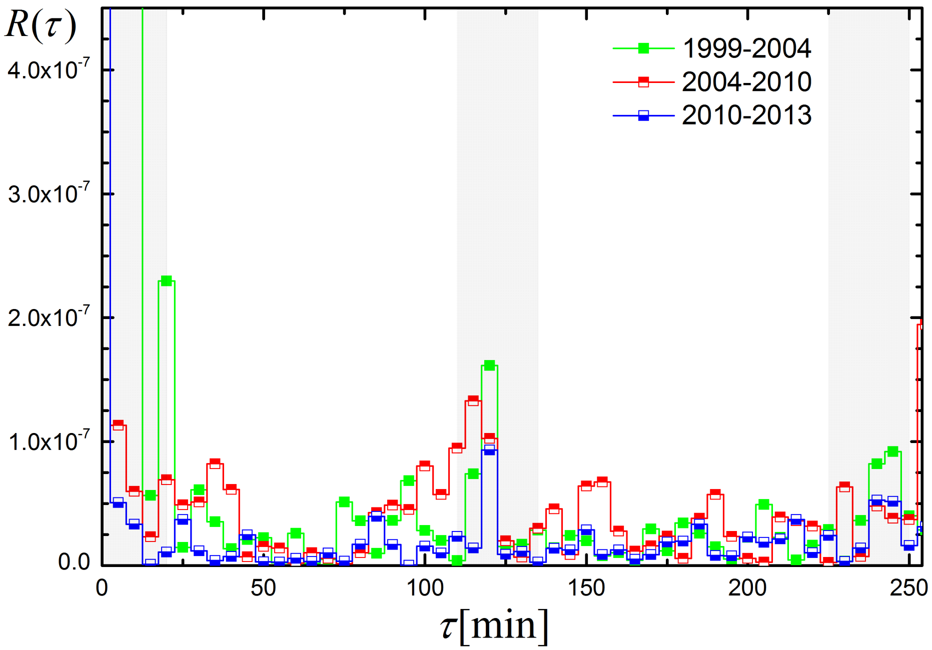
$M=20$,
$kT=\hbar=\gamma=1$

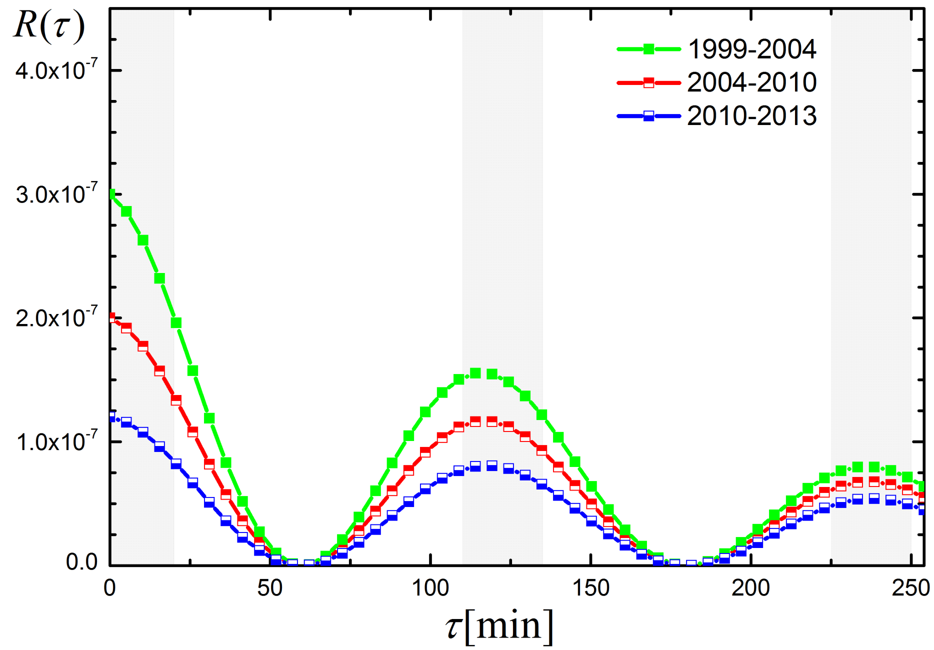
$\langle X^2 \rangle=\langle P^2 \rangle=\hbar/2$, $\langle XP+PX \rangle=0$.

The kurtosis $\kappa_x^4(t)$ of qBm has an **exponential decrease** as the actual kurtosis of the stock index (Shanghai Composite Index) does.

Data Analysis

Kurtosis and Autocorrelation of qBm(m)





Fitting autocorrelation function $R(\tau)=(\xi e^{-\eta\tau/2}\cos\Omega\tau)^2=\frac{1}{2}\xi^2 e^{-\eta\tau}(\cos 2\Omega\tau+1)$,

Non-Markovian master equation of qBm:

$$\frac{d}{dt}\rho_A(t)=-\frac{i}{\hbar}[H_A,\rho_A(t)]+\mathcal{K}(t)\rho_A(t),$$

$$\mathcal{K}(t)\rho_A(t)=-\frac{i\gamma}{\hbar}[X,\{P,\rho_A(t)\}]-\Delta(t)[X,[X,\rho_A(t)]]+\Lambda(t)[X,[P,\rho_A(t)]].$$

Data Analysis
Additional Less-Colored Noise

Non-Stationary → Scaling in Detrended Fluctuation Analysis [1]

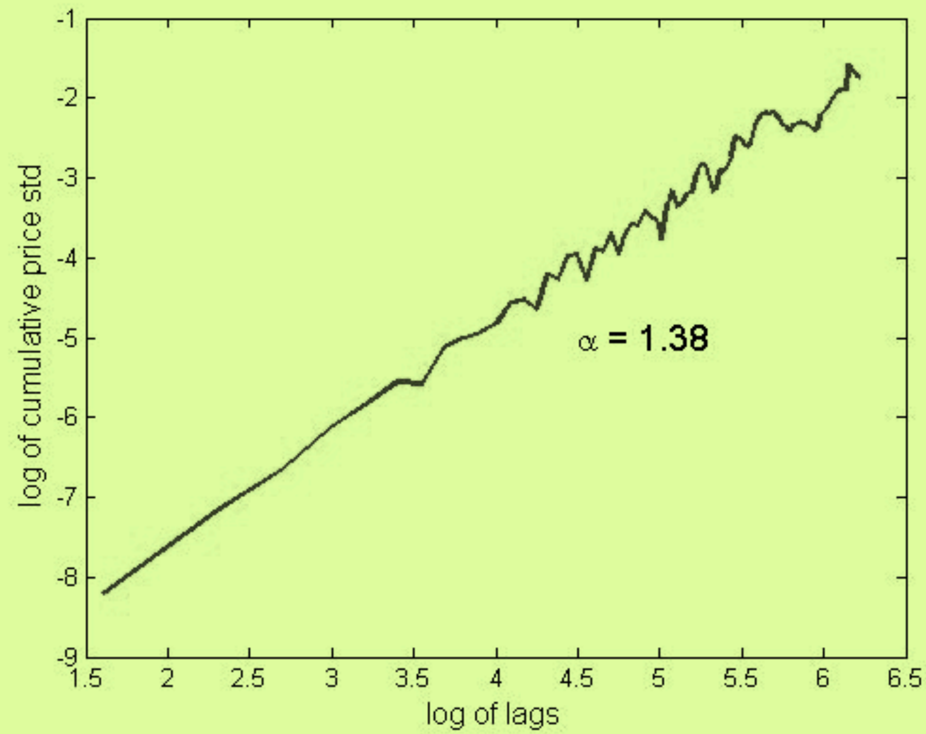
$$F(L)=\left[\frac{1}{L}\sum_{j=1}^{L}(Y_j-ja-b)^2\right]^{\frac{1}{2}}.$$

$$F(L)\propto L^{\alpha}$$

α=1/2: white noise;

α=1: pink noise;

α=3/2: Brownian noise;



- The propagating noise is **less-colored** than Brownian noise.
- There exists **additional** less-colored intrinsic noise (simplest example: additional white noise)

[1] H. Kantz and T. Schreiber, *Nonlinear Time Series Analysis* (Cambridge University Press, 2004).

Summary

Summary

Take-home messages

- Irrationality: additional persistent fluctuations from quantum uncertainty relation.

- Quantum open system dynamics: fat-tail phenomena and non-Markovian behaviors.

- Detrended fluctuation analysis: less-colored intrinsic noise in propagation.

Special thanks to…

- H. E. Stanley, Boston University
- J.-W. Zhang, Peking University
- H. Guo, Peking University
- B. E. Baaquie, National University of Singapore
- J. P. Huang, Fudan University
- P. Chen, Peking University

… and many others