Lecture Feb 12

Dynamical phenomena in single and interacting networks

Antonio Majdandzic
Boston University

***Advisor*:**
H. E. Stanley

Collaborators:

B. Podobnik	L. Braunstein
S. Havlin	I. Vodenska
S. V. Buldyrev	C. Curme
D. Kenett	S. Levy-Carciente
T. Lipic	D. Horvatic

Outline

1. Introduction: failures & recoveries

2.1 **Single networks** phase diagram

2.2 Finite size effects

3.1 **Interacting networks** phase diagram

3.2 Finite size effects

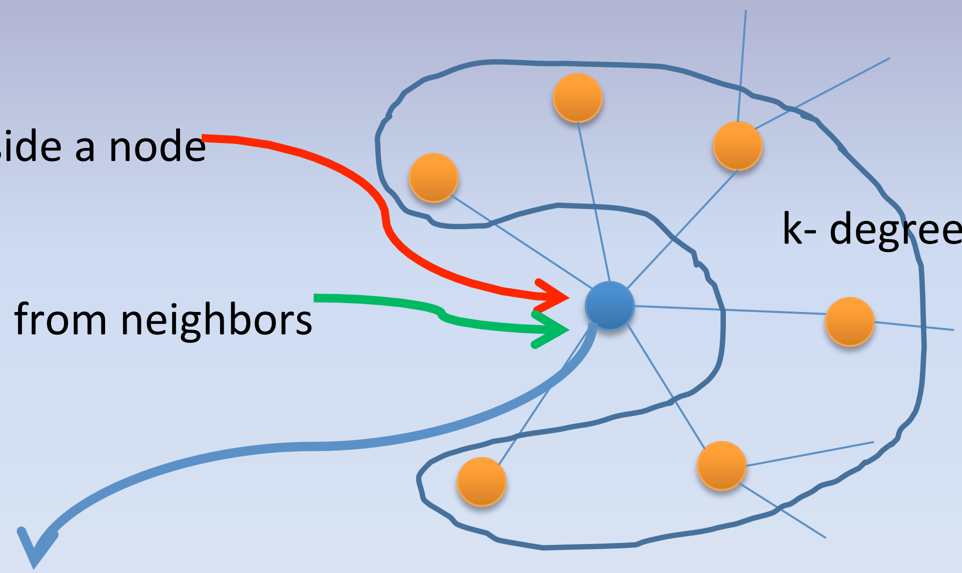
A network model with failures and recoveries.

• Each node in a network can be **active** or **failed**.

• We suppose there are **TWO possible reasons for the nodes' failures**: INTERNAL and EXTERNAL.

1. INTERNAL failure: intrinsic reasons inside a node

2. EXTERNAL failure: damage "imported" from neighbors

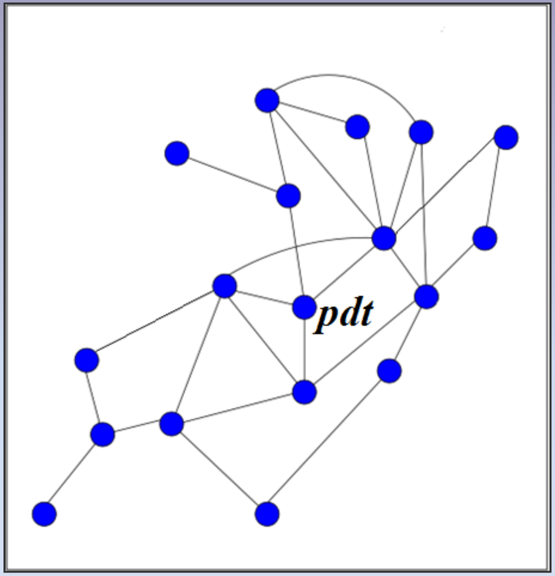


RECOVERY: A node can also **recover** from each kind of failure.

LET'S SPECIFY/MODEL THE RULES.

1. INTERNAL FAILURES

p- rate of internal failures (per unit time, for each node). During interval *dt*, there is probability **p***dt* that the node fails.



Recovery: A node *recovers from* an *internal failure after a time period* **τ** .

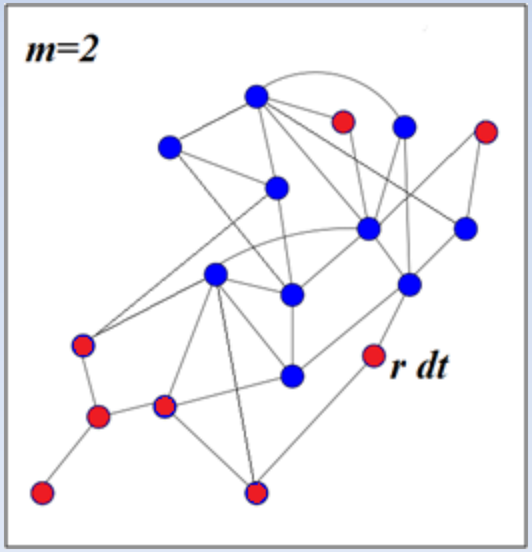
2. EXTERNAL FAILURES – if the neighborhood of a node is too damaged

IF: “CRITICALLY DAMAGED neghborhood”: **less than or equal to m active neighbors, where m is a fixed treshold parameter.**

THEN: There is a probability $r\,dt$ that the node will experience externally-induced failure during *dt*.

r - external failure rate

A node recovers from an external failure after time **τ** *′.*



FAILURE TYPE	RULE	RECOVERY
Internal failure	With rate p on each node	After time τ
External failure	IF(<= m active neighbors) THEN Extra rate r on each node	After time τ'

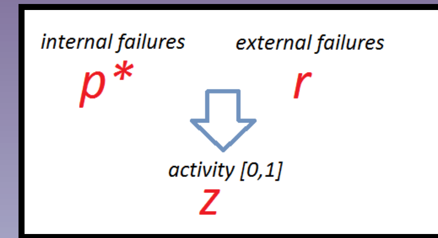
Out of these 5 parameters, we fix three of them:
m=4, $\tau=100$ *and* $\tau'=1$.

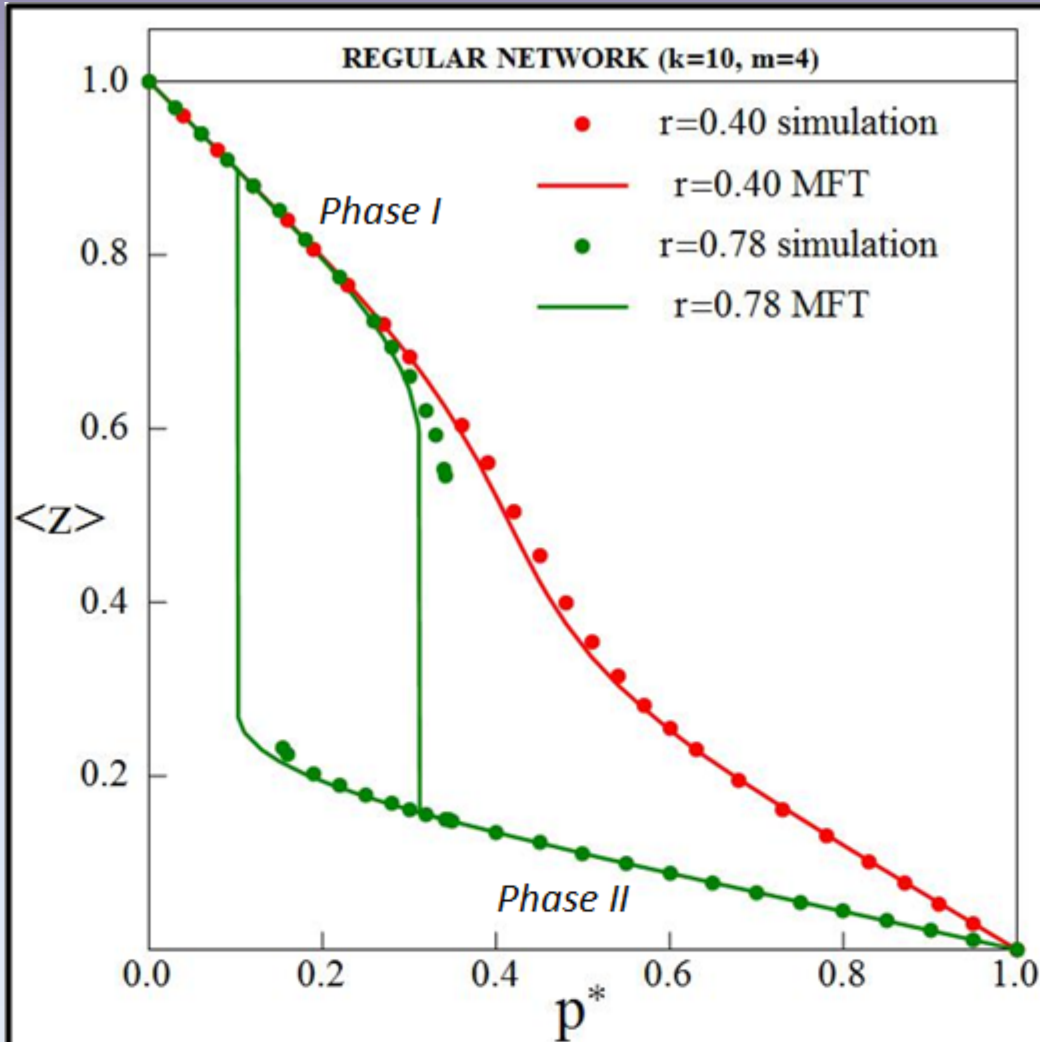
We let (p,r) to vary.

It turns out it is convenient to define $p^*=\exp(-p\tau)$.
So we use (p*,r) instead of (p,r).

We measure activity Z of the network as a function of (p*,r).

Model simulation [Random regular networks]



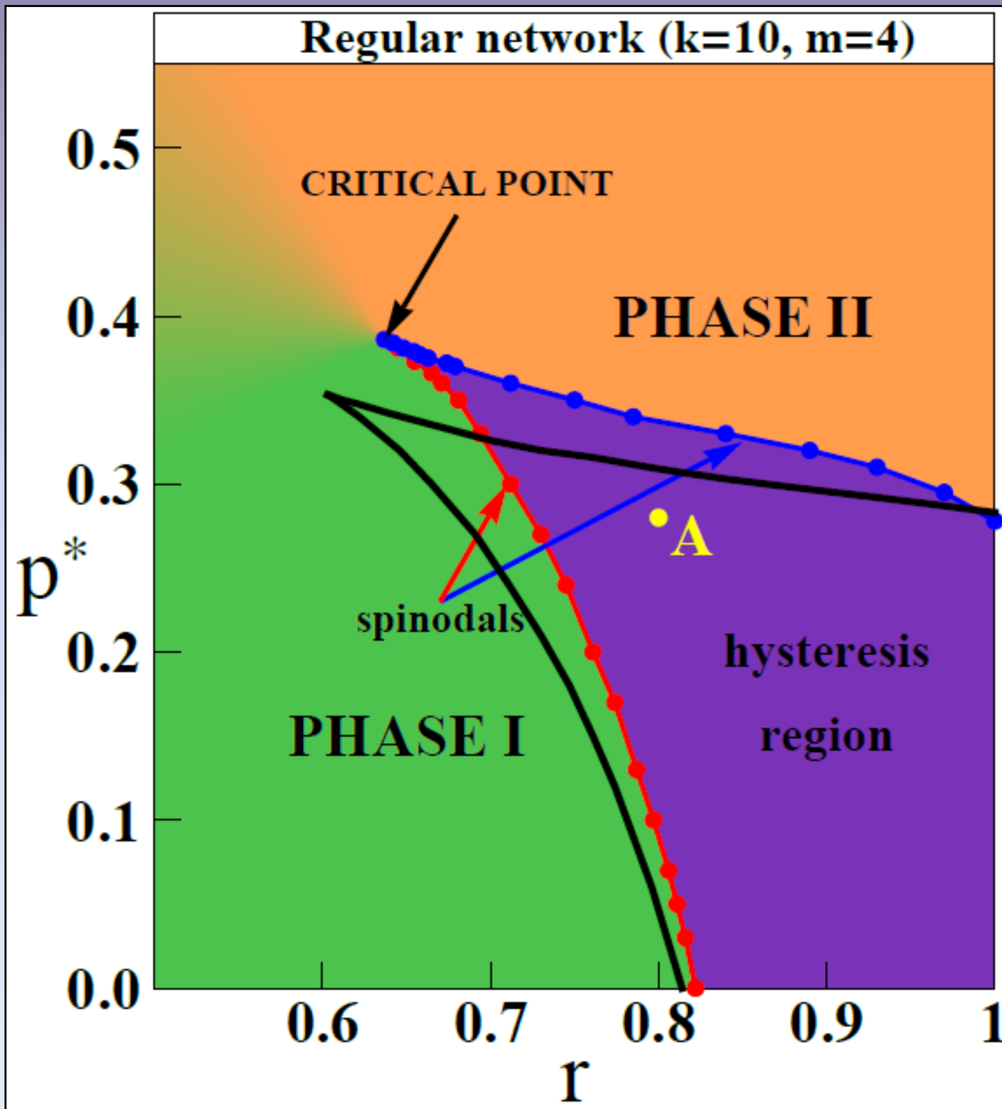


<z>- average fraction of active nodes (Z fluctuates)

We fix r, and measure <z>(p*)

For some values of *r* we have a hysteresis loop.

Phase diagram (single network, random regular)



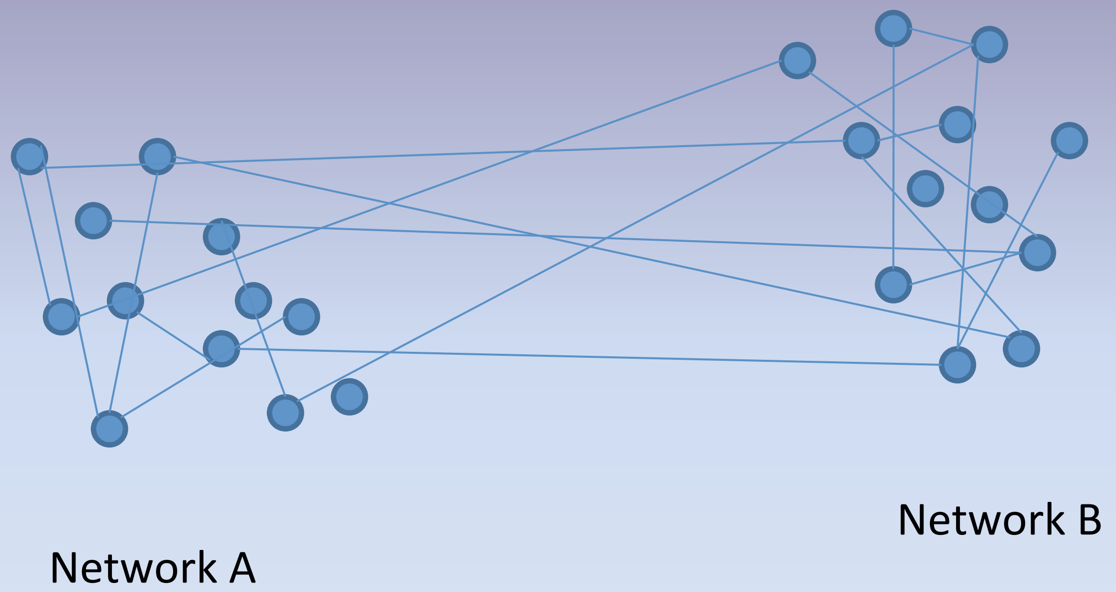
GREEN; High activity Z
ORANGE: Low activity Z

In the hysteresis region both phases exist, depending on the initial conditions or the memory/past of the system.

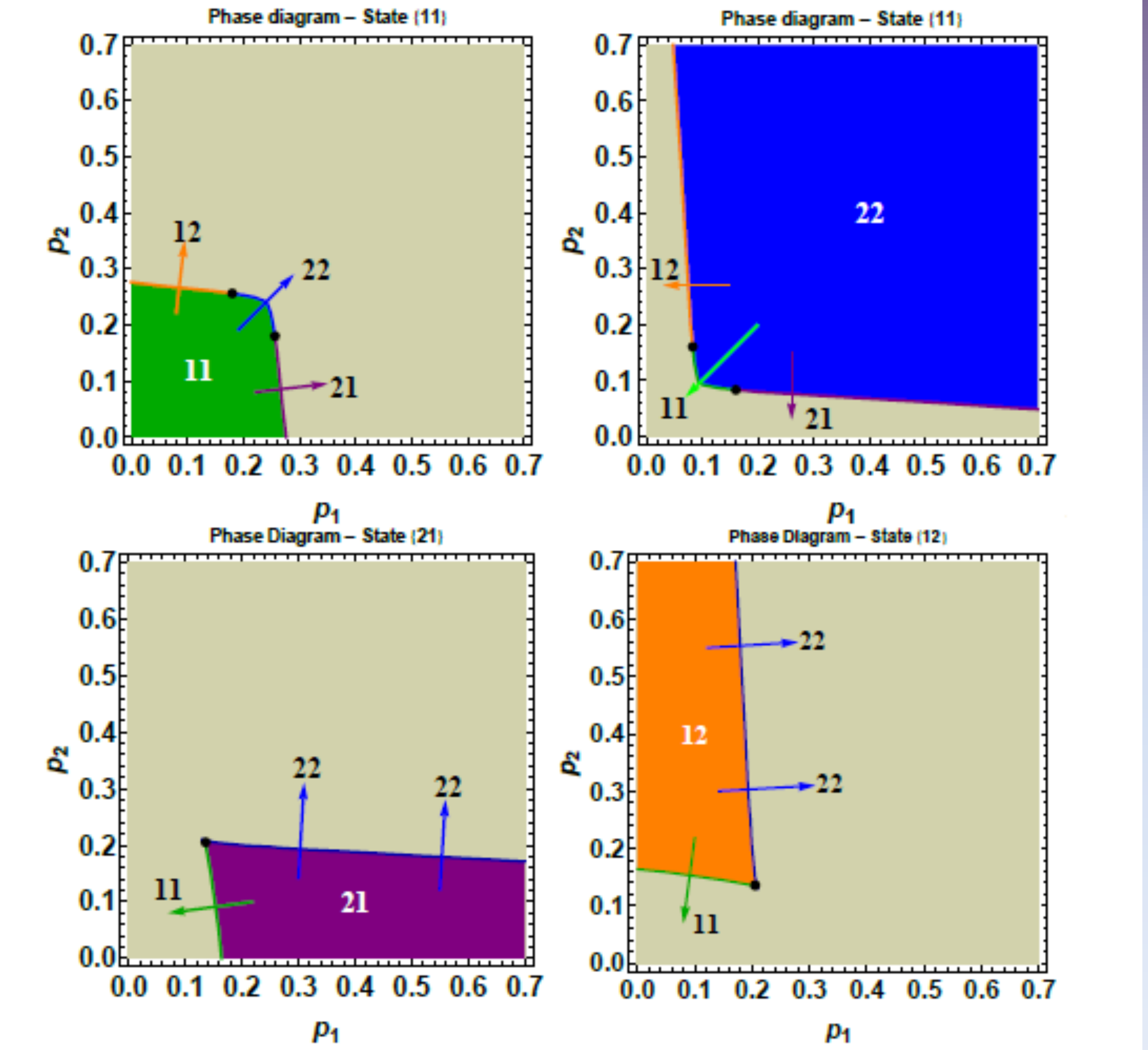
Blue line: critical line (spinodal) for the abrupt transition I→ II

Red line: critical line (spinodal) for the abrupt transition II→I

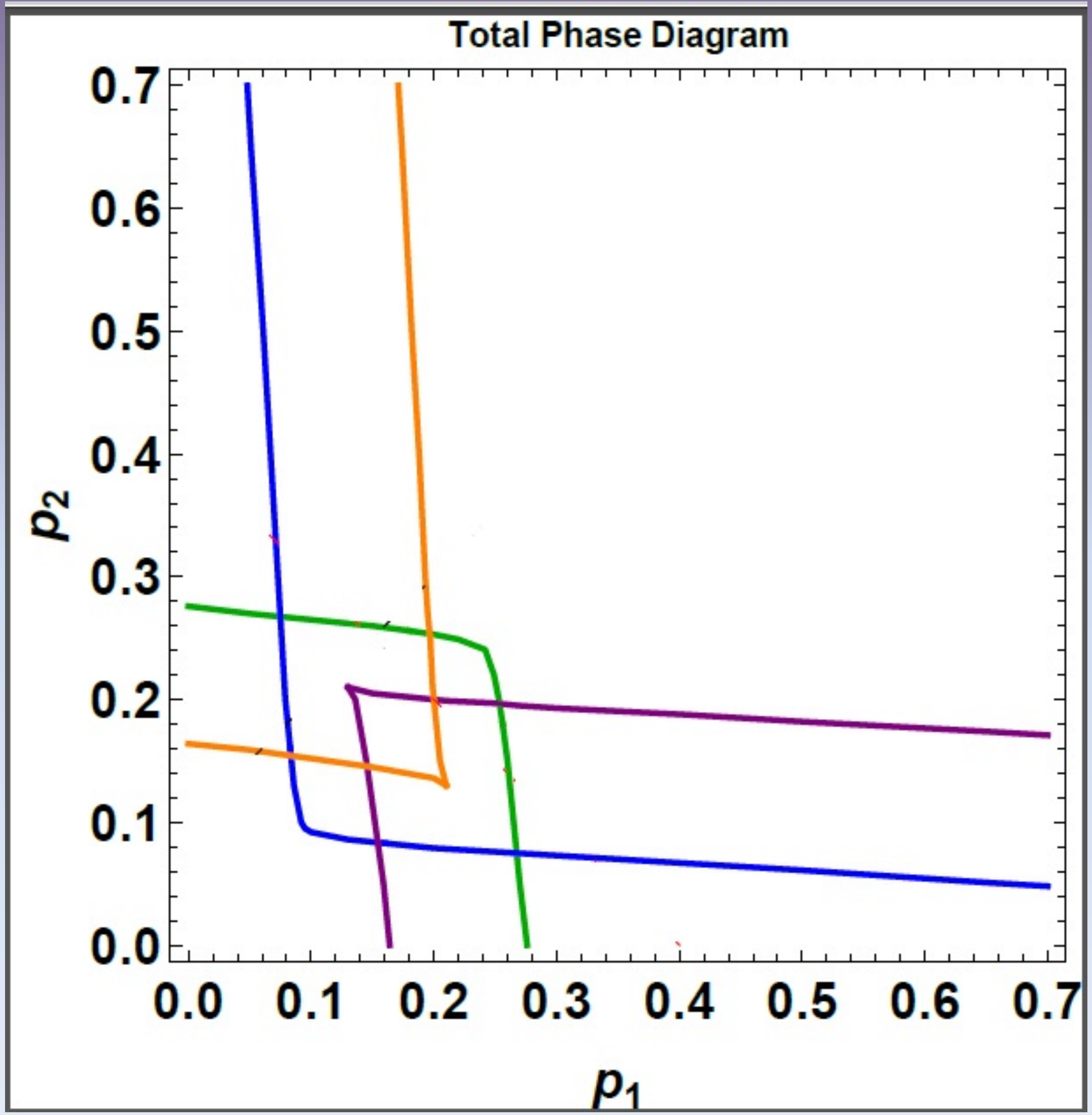
3.1. Interacting networks



FAILURE TYPE	RULE	RECOVERY
Internal failure	With rate $\boldsymbol{p}$ on each node	After time $\boldsymbol{\tau}$
External failure	IF(<= m active neighbors) THEN Extra failure rate $\mathbf{r}$	After time $\boldsymbol{\tau}'$
Dependency failure	IF(companion node from the opposite network failed) THEN Extra failure rate $\mathbf{r_d}$	After time $\boldsymbol{\tau}''$

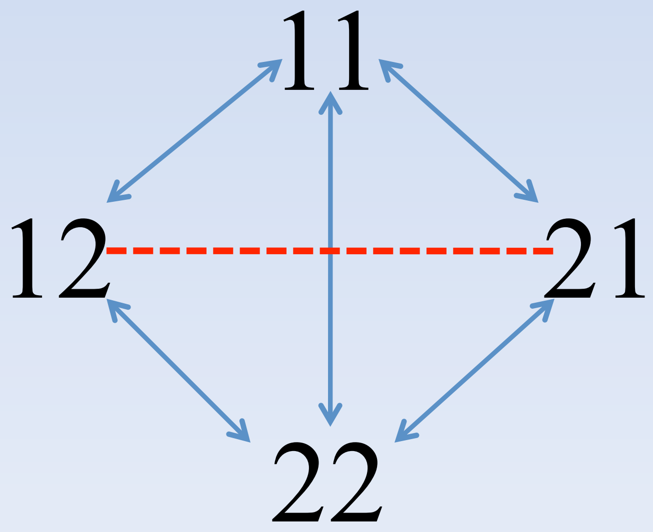


Phase diagram – State |11⟩
11
12
22
21
Phase diagram – State |11⟩
22
12
11
21
Phase Diagram – State |21⟩
21
22
22
11
Phase Diagram – State |12⟩
12
22
22
11
p_1
p_2


Total Phase Diagram
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
p_2
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7
p_1

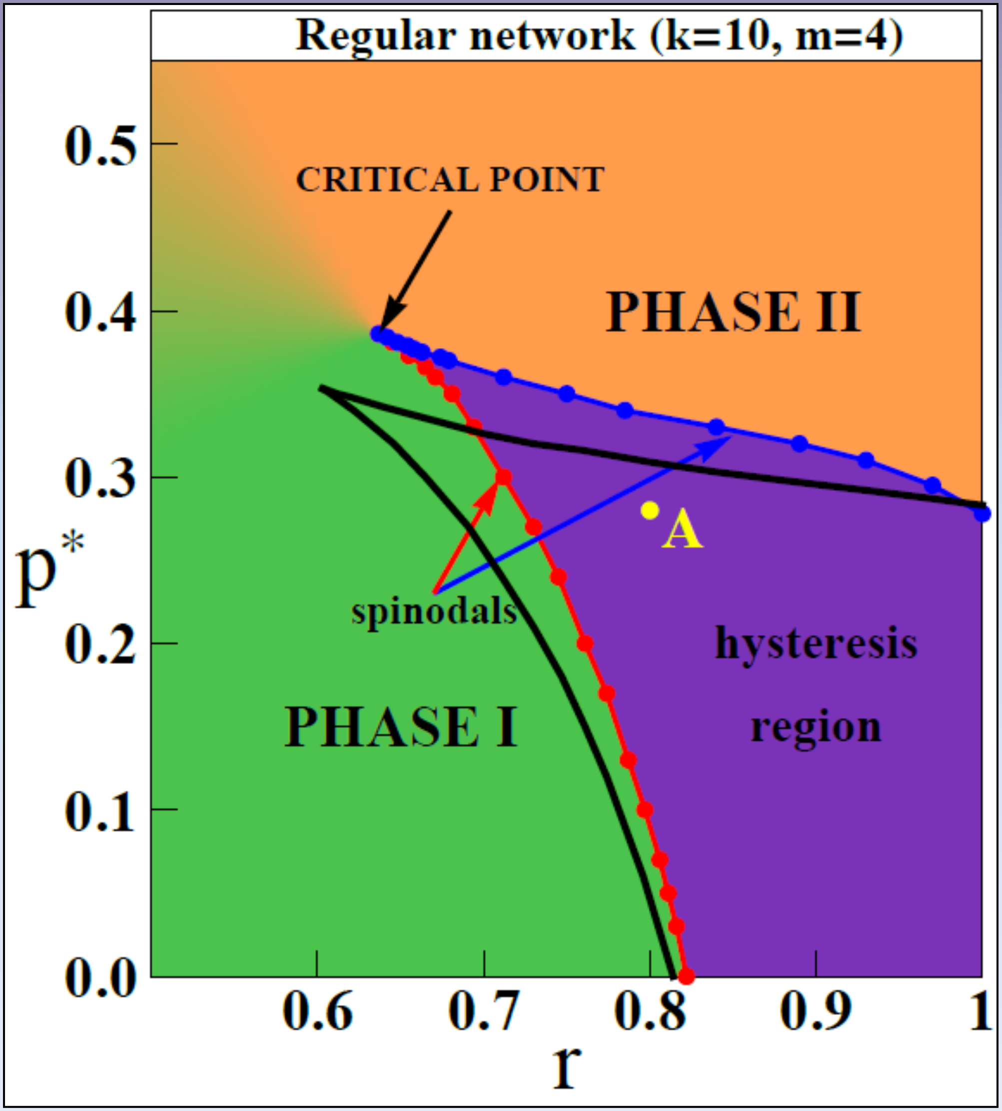
Elements of the phase diagram

- 2 critical points
- 4 triple points
- 10 allowed transitions
- 2 forbidden transitions



Using network representation to model phenomena in finance

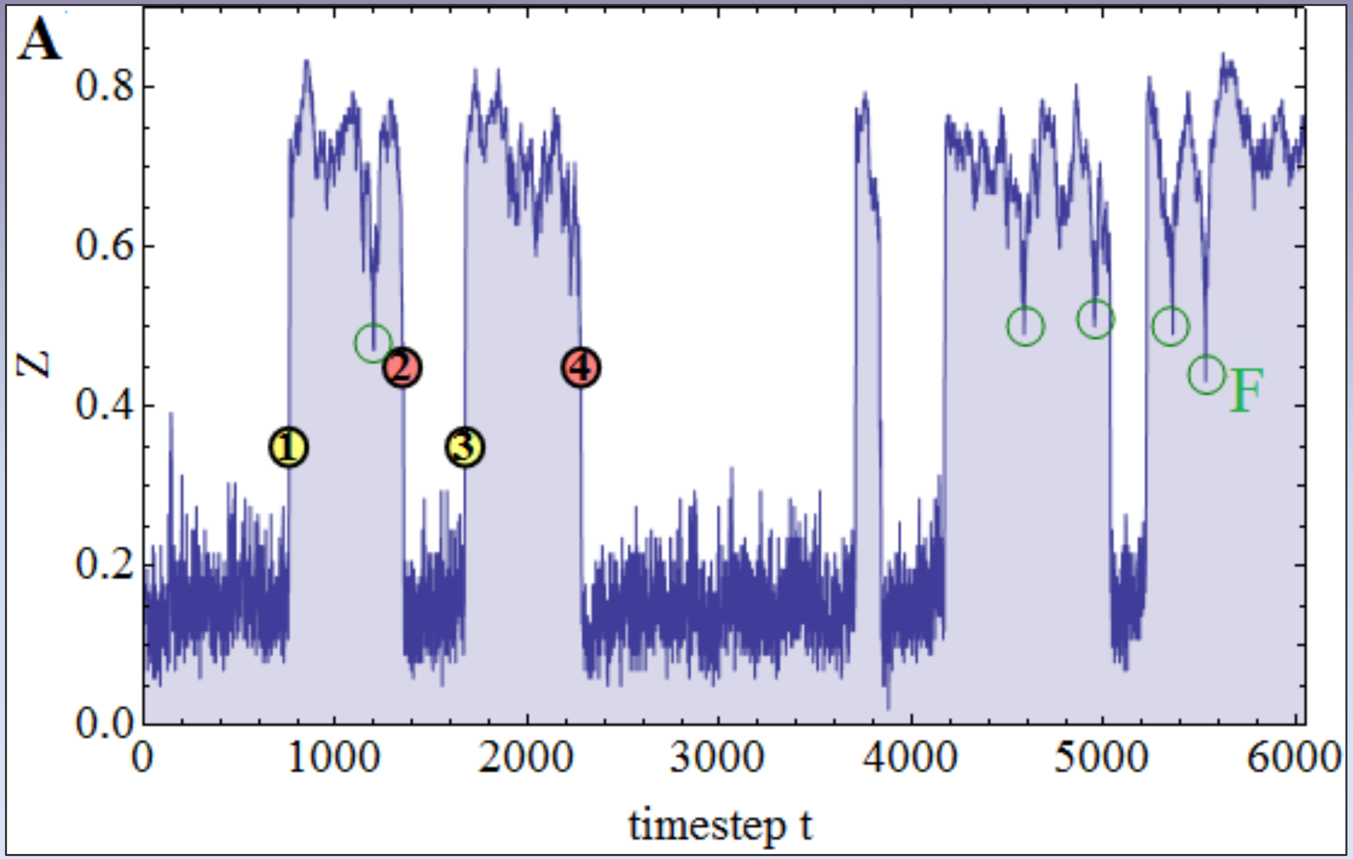
Let's pick point A, take a small system N=100, and run the simulation



Finite size effects

Sudden transitions

Is there any forewarning?

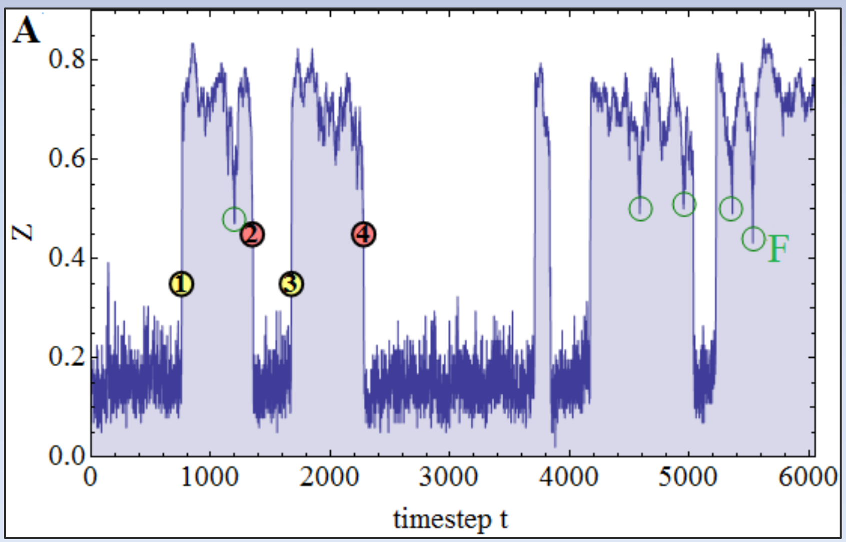


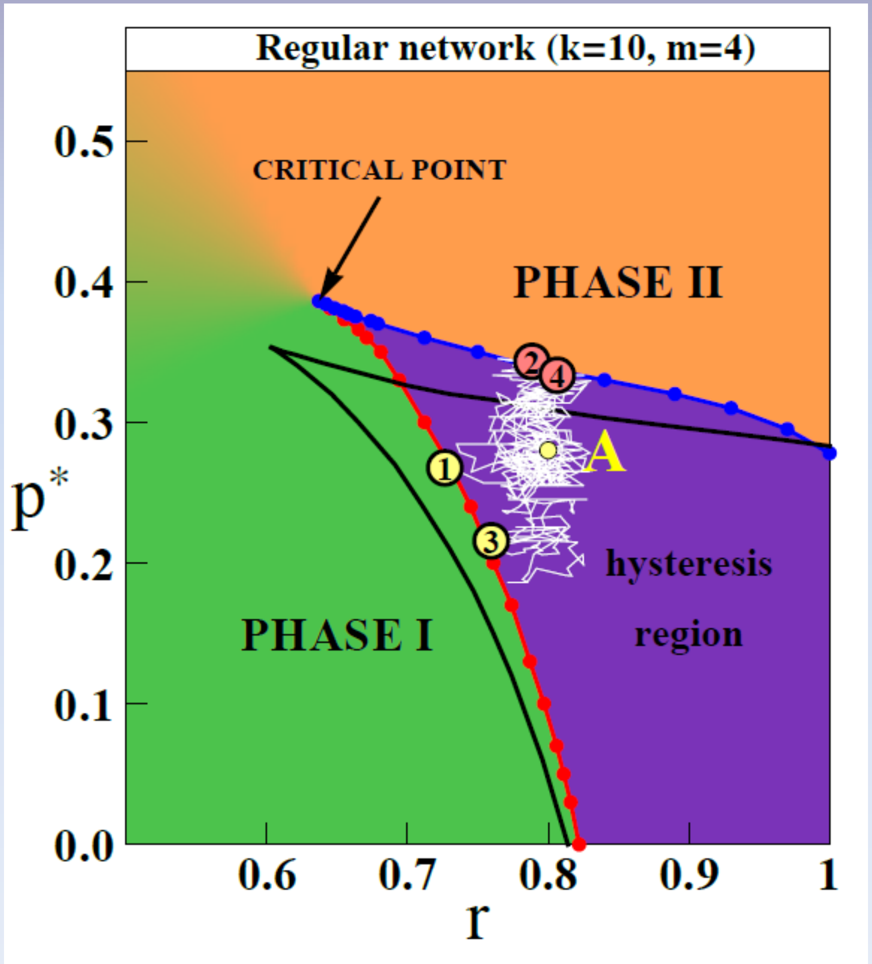
(Remember : Z = Fraction of active nodes)

It turns out it can be predicted (to some extent).

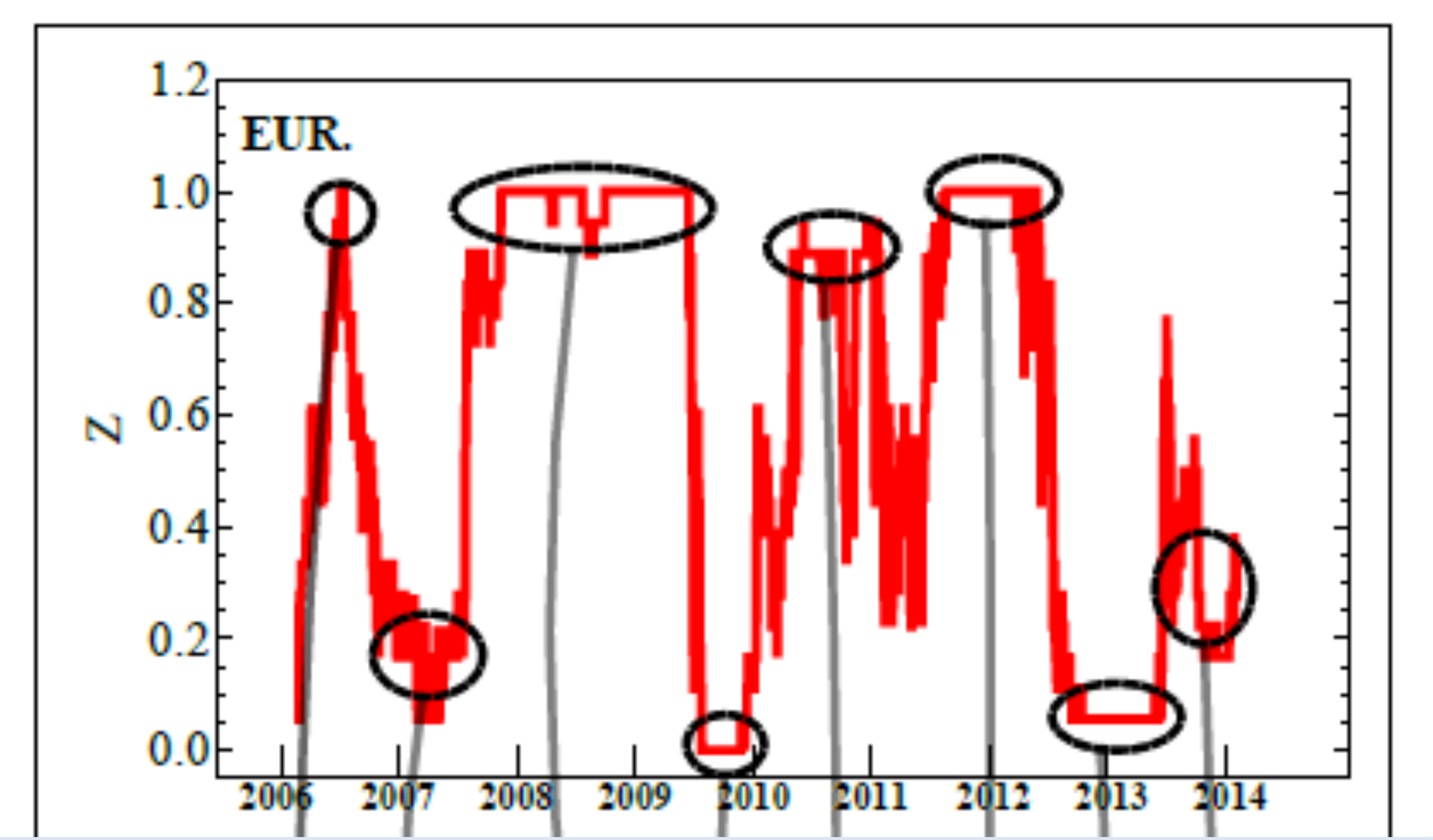
Trajectory $(r_\lambda(t),\ p_\lambda{}^*(t))$ in the phase diagram (white line, see below).

The trajectory crosses the spinodals (critical lines) interchangeably, and causes the phase flipping.



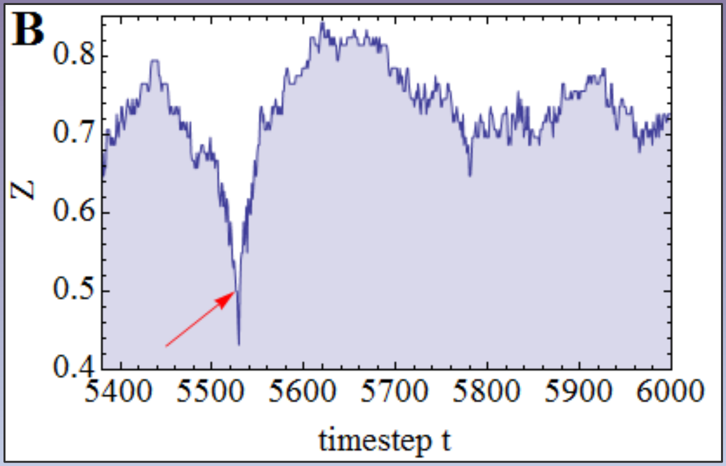


CDS European network, in time (2006-2014)



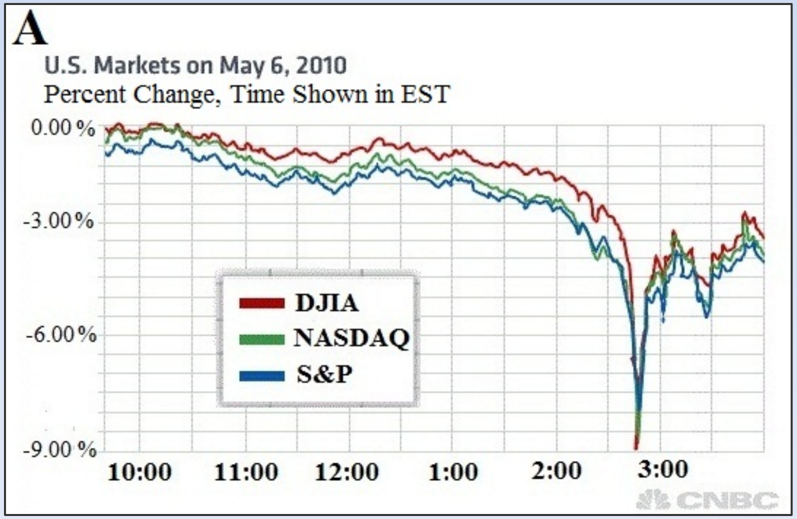
Model also predicts the existance of “flash crashes”.

Explanation: Unsuccessful transition to the lower state.



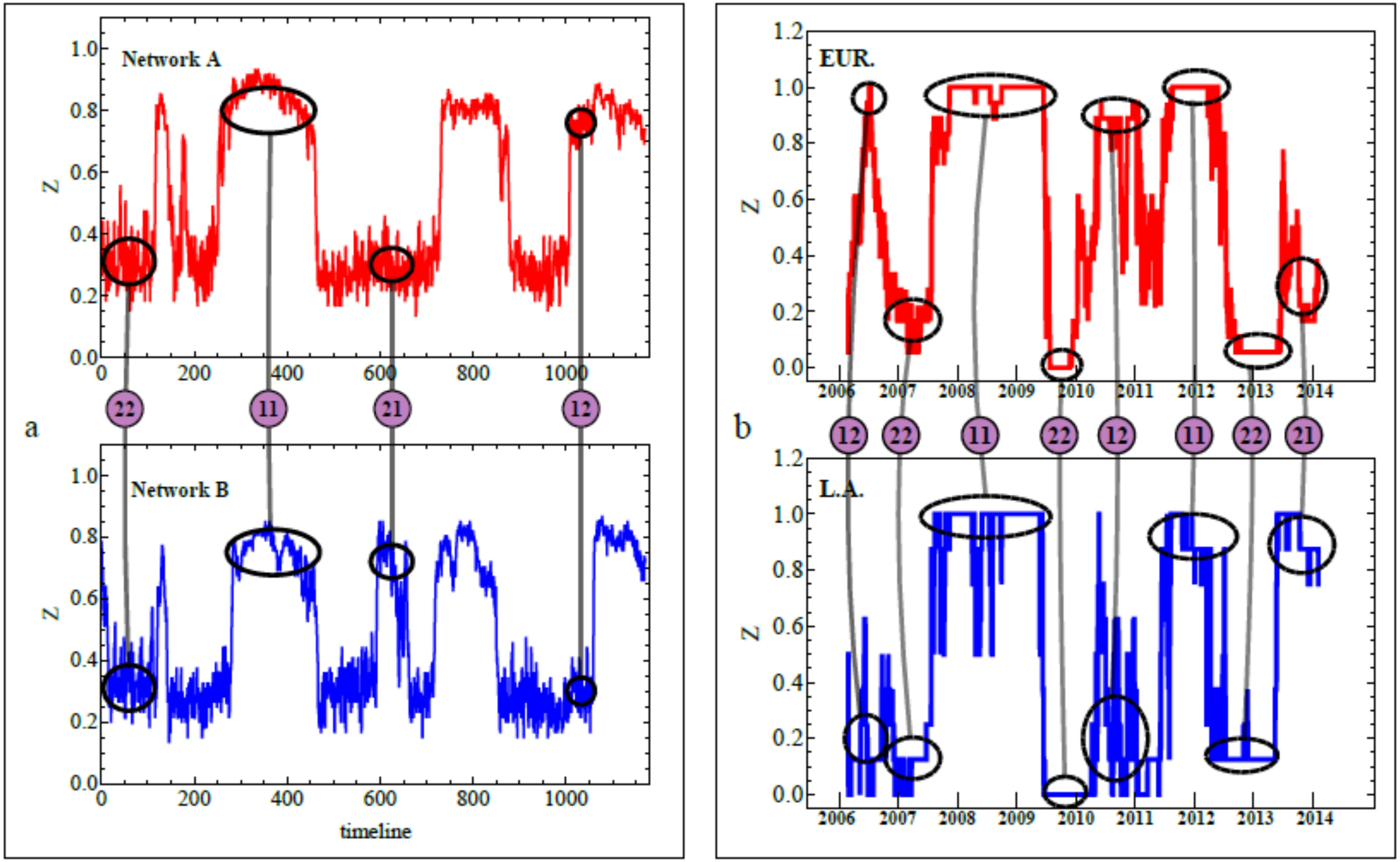
Real stock markets also show a similar phenomenon.

Q: Possible relation?



“Flash Crash 2010”

Simulated and real interacting networks: CDS networks



6: Collective states in simulated and real interacting networks.