The Effect of Failure Sites in the Asset Exchange Model

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We introduce failure sites into the asset exchange model to test its stress response. We see that the system recovers very quickly, and reintroduces known behavior in the long-time limit. We introduce a flat and sales tax and determine that flat tax is more influential as the system recover from failed sites.

I. INTRODUCTION

It has frequently been shown that simple models yield great insight into physical phenomena. The simple model we use here is the asset exchange model (AEM) that is used as a simple model to understand wealth inequality. In the AEM, an equal amount of wealth is initially distributed to N traders. We then choose two traders, i and j, at random and allow them to trade. The trading mechanism takes a fraction of the poorer agents wealth and gives both traders equal probability of receiving that amount. We define the exchange wealth to be

$$\delta w = \alpha \min(w_i, w_j), \tag{1}$$

where $\alpha \in [0, 1]$ is a parameter giving the fraction of the poorer agents wealth will be exchanged.The losing trader loses a fraction of the poorer agents wealth while the winning agent gains that same amount. As the system is run for a long period of time, the wealth condenses into one agent while the other agents end up with no wealth as $t \to \infty$ [1]. It has been shown that growth will lead to a phase transition that results in a rescaled steady state [2]. Moreover, the inclusion of a flat and sales tax with redistribution leads to a phase transition if the redistribution favors the poor agents [3].

In this paper, we modify the AEM by introducing failed sites with negative wealth that must be "bailed out" by the other agents. In our simulations we set the agents with a starting wealth of $w_i(0) = 1$. We define a timestep to be N trades for N traders. After 10 timesteps have passed, we introduce an certain percentage of failed sites given by the parameter Ω that have negative wealth. This is in the spirit of quenching in statistical thermodynamics. We are interested in seeing how the system recovers from these failure sites, so we make our measurements 10 trades after the failure. Since the wealth in the system is not conserved due to the failure, we defined the rescaled wealth to be $\tilde{w}_i = \frac{w_i}{W(t)}$ where $W(t)$ is the total wealth in the system.

We first ask how the system recovers after site failure. Figure 1 shows hows the rescaled wealth of the system changes from before the failure to after. We see that after 10 trades the system has completely recovered, and wealth condensation is occurring. This implies a quick recovery from this sudden shock in the asset exchange model.

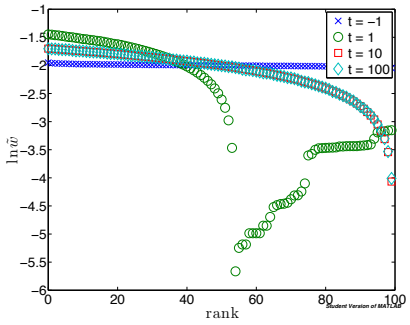

FIG. 1: Rescaled wealth as a function of rank for $\Omega = 0.99$ and $\alpha = 0.01$. Shows the system recovers from the shock after 10 trades.

We are interested in how failing out sites affects how an agents rank is correlated with his rank in the past. We introduce the rank correlation function, defined as:

$$C(t) = \frac{\sum_i [R_i(t) - R(t)][R_i(0) - R(0)]}{[\sum_j (R_j(t) - R_j(t))^2][\sum_k (R_k(0) - R_k(0))^2]}, \tag{2}$$

Where $R_i(t)$ is the rank of the ith agent at time t, and $R(0)$ is the ensemble average of $N/2$. A correlation of 1 implies that the agents rank is completely determined with his rank in the past. A correlation of 0 implies an agents rank is completely uncorrelated with the past. In Fig. 2 we show the correlation function where the shock occurs at 10TS. The results show that increasing the trading parameter results in a steep decline of an agents correlation after the site are failed.

The relationship between the trading parameter and the failure rate is shown in Fig. 3. The result is that the rich agents are only weakly dependent on the failure rate, but are highly dependent on α. Whereas the poor agents prefer low values of α and values of Ω up to $\Omega = 0.5$. In addition, there are some distributed regions for high α where the poor agents have less wealth.

II. FLAT TAX

We now introduce a flat wealth tax into the system. We take a fraction of all agents wealths', $\psi \in [0, 1]$, and redistribute it equally to all agents. So every trade the

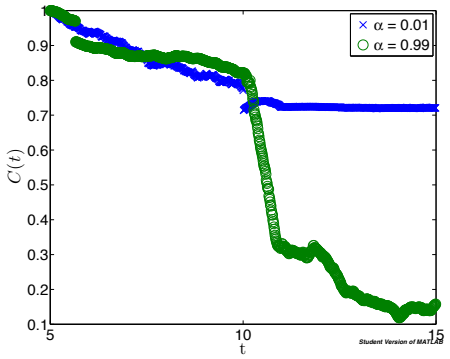

FIG. 2: Correlation function over time with $\Omega = 0.99$. Increasing the trading parameter α results in a less correlated system.

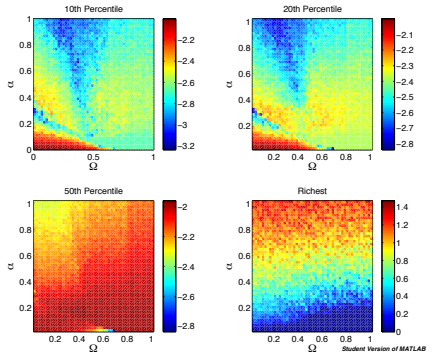

FIG. 3: Log of the rescaled wealth as a function of α and Ω four trades after site failure. Rich agents are highly dependent on α but depend very little on Ω.

ith agent loses an amount ψw_i, but gains back wealth $\psi W(t)/N$ where $W(t)$ is the total wealth in the system at time t. As $\psi \to 1$ the system will become more equal, with $\psi = 1$ resulting in a completely equal distribution. In Fig. 4 we look at the wealth distribution for an applied $\psi = 0.01$, $\Omega = 0.99$, and $\alpha = 0.10$ 1 trade after the failure. The result shows that increasing the flat tax rate will result in a more equal distribution, but for low values of the flat tax the system does not differ in behavior from the case without a flat tax.

Figure 5 shows the full range of the relationship between ψ and Ω. We see that the rich agents perfect low failure rates and a low flat tax rate. The behavior of the poor agents demonstrates that except for high failure rates the flat tax overpowers the failed sites.

III. SALES TAX

We apply a tax on the exchange wealth between agents, taking an amount $\beta \in [0, 1]$ from the exchange wealth and redistributing it equally among agents. In Fig. 6 we see

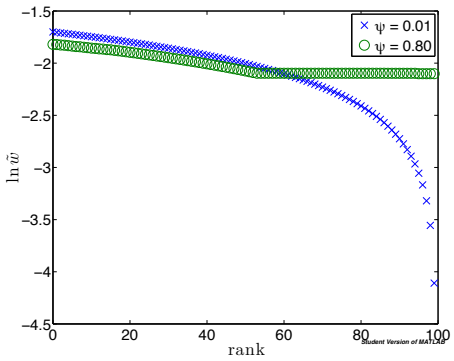

FIG. 4: Log of the rescaled wealth for $\alpha = 0.10$, $\Omega = 0.99$, and $\psi = 0.01$ one trade after failure. Flat tax results in greater separation between rich and poor agents after failure.

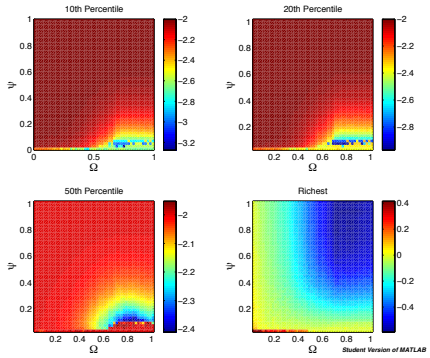

FIG. 5: Log of the rescaled wealth for $\alpha = 0.10$ five trades after failure.

that the wealth distribution one trade after failure does not depend on the sales tax rate.

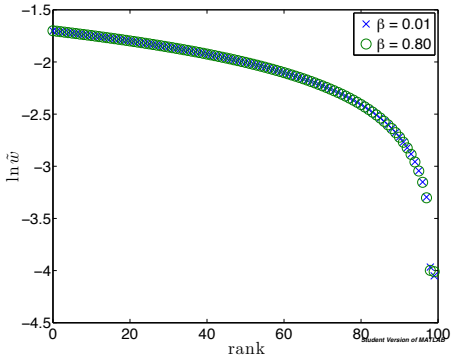

FIG. 6: Log of the rescaled wealth for $\alpha = 0.10$ and $\Omega = 0.99$ one trade after failure. Rescaled wealth distribution is independent of the sales tax.

The total relationship between the sales tax and the failure rate is given in Fig. 7. We confirm that the agents

are only weakly dependent on the sales tax, β for all values of Ω.

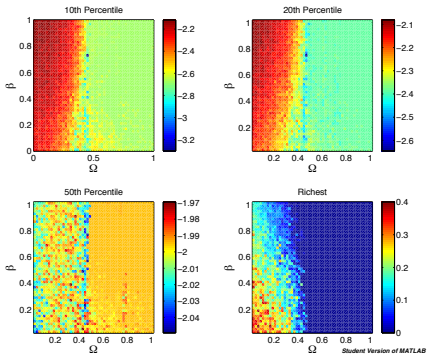

FIG. 7: Log of the rescaled wealth for $\alpha = 0.10$ as a function of β and Ω four trades after site failure.

IV. FLAT AND SALES TAX

Applying both the flat and the sales tax allows us to investigate their effect after a shock. We see in Fig. 8 that the combined flat and sales tax one trade after failure reflect the fact that the redistributive sales tax is only weakly effective after a shock. The flat tax is a more important quantity as the system recovers. In the long-time limit after the shock, the system recovers completely and reproduces the expected behavior for the flat and sales tax.

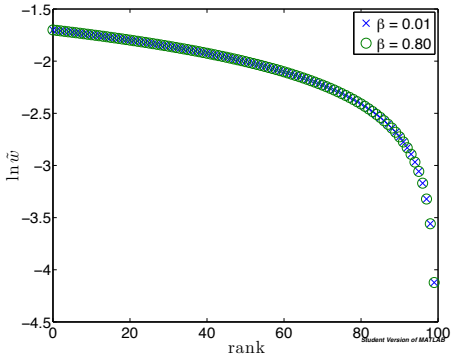

FIG. 8: Log of the rescaled wealth for $\alpha = 0.10$, $\psi = 0.01$, and $\Omega = 0.99$ one trade after failure.

V. CONCLUSION

Introducing failed sites to the asset exchange model leads to a quick recovery after 10 trades. In the long-time limit after the failure, wealth condensation will reappear. Applying a flat tax shows the poor agents lose more wealth for low values of the flat tax with high values of the failure rate shortly after the shock. A high sales tax benefits the poorer agents in the system, while a low sales tax benefits the richer agents. However the benefit gained is of order zero, showing that the sales tax is not strong in affecting the wealth distribution after the failure. As we let the system run after the failure, the phase transition that appears in the model without failed sites appears. We conclude that the introduction of failed sites in the asset exchange model does not affect the long-term behavior of the model, even with taxation. Moreover, we see that the system recovers very quickly after only a few trades and that the flat tax is more influential than the sales tax as the system recovers.

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